Modification of CST Airfoil Representation Methodology

By Brenda Kulfan

This note presents a simple but highly effective modification to the CST airfoil representation method in the region of the airfoil nose. The Class function Shape function Transformation Method was shown in the references to be a very effective process to mathematically represent airfoils as well as other classes of 2D, 3D and axisymmetric geometries. It was shown that the upper and lower surface of an airfoil could be described as the simple product of a class function times a shape function.

For a round nose airfoil the class function is: $$C(\psi) = \psi^{0.5}(1-\psi) \tag{1}$$

ψ is the chordwise station x/c

It was convenient for most cases to represent the shape function using Bernstein polynomials. Each term of the selected Bernstein polynomial defines a component shape function, $S_i(\psi)$, such as:

$$S_i(\psi) = K_i \cdot \psi^i \cdot (1-\psi)^{N-i} \tag{2}$$

Where K_i is a binomial coefficient defined as $$K_i = \frac{N!}{(N-i)!\, i!} \tag{3}$$

where N is the order of the binomial coefficient and i = 0 to N

Each of the component shape functions when multiplied times the class function defines a component airfoil shape:

$$\zeta_i(\psi) = C\{\psi\} \cdot S_i \cdot (\psi) \tag{4}$$

Where ζ = z/c

The figure below shows the component shape functions and the corresponding component airfoil shapes for a Bernstein polynomial of order 4.

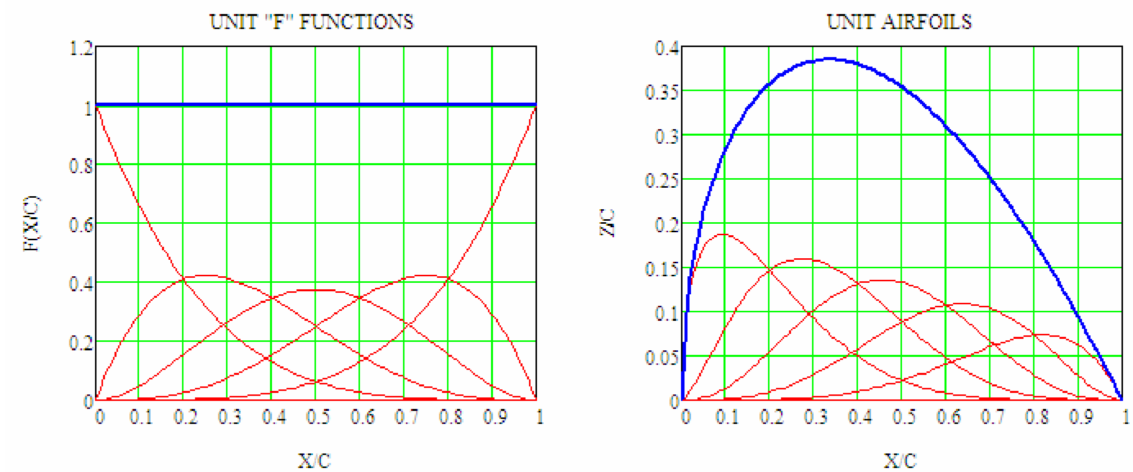

Fig. 1. Composite Shape Functions and Composite Airfoils for a BPO = 4 Unit Shape function Airfoil

The surface coordinates of an arbitrary airfoil expressed in terms of the composite airfoils is given as:

$$ZoC(\psi) = \sum_{i=0}^{N}\left(A_i \cdot \zeta_i(\psi)\right) + \psi \cdot \Delta\zeta_{TE} \tag{5}$$

The A_i are "to be determined" scaling coefficients.

A similar equation applies to both the upper and lower surface of an airfoil.

Previous studies have shown that all classes of symmetric and cambered airfoil geometries can be very closely matched by this methodology.

The criteria used to establish the quality of the approximating geometry included statistical measures such as coefficient of determination and standard deviations together with the following defined “exactness Conditions:

Manufacturing “Exact”

The differences between all of the actual airfoil coordinates and the corresponding approximating airfoil coordinates are less than wind tunnel or full scale manufacturing tolerances.

The definition based on an equivalent 2.7% scale model corresponds to:

For X/C = 0 to 20% : |Airfoil “Z” – Approximate “Z”| < 0.003 inches

For X/C > 20% : |Airfoil “Z” – Approximate “Z”| < 0.006 inches

Measurement “Exact”

The differences between all the actual airfoil coordinates and the approximating airfoil coordinates are less than typical wind tunnel measurement capability.

This corresponds to |Airfoil “Z” – Approximate “Z”| < 0.001 inches

Aerodynamically “Exact”

The differences between the calculated aerodynamic force, pressure and moment data for the actual airfoil and the approximate airfoil are less than some specified experimental data measurement tolerances

Our definition is |”Actual” Airfoil CD – “Approximate” Airfoil CD| < 0.00005 (1/2 Count)

The slopes, second derivatives and curvature computed for the approximate geometries were also compared with those determined from the actual geometry.

These exactness criteria and the aforementioned evaluation approach is consistent with “Brenda’s rules of approximation” shown below

Brenda’s Rules of Approximation

Rule 1: An Approximation is only Approximate

Rule 2; A functionally exact approximation is not an approximation
Corollary 2-1: An Approximation is never functionally exact

Rule 3: A mathematically correct approximation is a good thing although not absolutely necessary.
A Mathematically correct approximation is defined as an approximation that has the dominant terms that typically occur at the edges of an approximation, correctly represented.

Rule 4: The residual differences between the actual and a least squares approximation will always exhibit oscillations about zero. The area of the positive oscillations tends to equal the area of the negative oscillations.

Rule 5: The oscillatory nature of the residual curve does not mean that either the actual or the approximation is an oscillatory function.

Rule 6: One can find a scale where the difference between the actual and any approximation to the actual “looks” large.

Rule 7: The quality of an approximation is determined by it’s intended usage

Rule 8: Specific goodness criteria to judge the quality of an approximation should be defined and must be related to the intended usage

Rule 9: A “Looks good Criteria” seldom is good enough

Rule 10: Know what’s important for a specific application– and be sure the important features are properly captured. Examples:
Subsonic Pressures are governed by surface curvature, therefore matching curvature is important
Supersonic pressures are governed by surface slopes, therefore matching slopes is important
Supersonic far field wave drag varies with surface ordinates, therefore matching surface ordinates is important.

A close examination of the equations 4 and 5 indicates that the airfoil coordination representation is a fractional series in the chordwise coordinate, ψ. As such, the mathematical representation does not contain a linear term near the nose. Consequently, this is not a mathematically correct approximation for some cambered airfoils as well as for some symmetric airfoils even though the approximate airfoil representation can be very close to the actual airfoil geometry and well within both manufacturing and measurement capabilities.

The classic definitions of the thickness and camber definitions for a general cambered airfoil are respectively:

$$\zeta_{thickness}(\psi) = \frac{\zeta_U(\psi) + \zeta_L(\psi)}{2} \quad \text{and} \quad \zeta_{camber}(\psi) = \frac{\zeta_U(\psi) - \zeta_L(\psi)}{2}$$

A very simple modification has been developed to remedy this situation that basically involves the addition of another dominate leading edge term to produce a sharp nose composite airfoil. As shown in the figure below, this is achieved by simply duplicating and reversing the last composite airfoil that has a sharp trailing edge and including the reversed sharp nose airfoil as an additional composite airfoil.

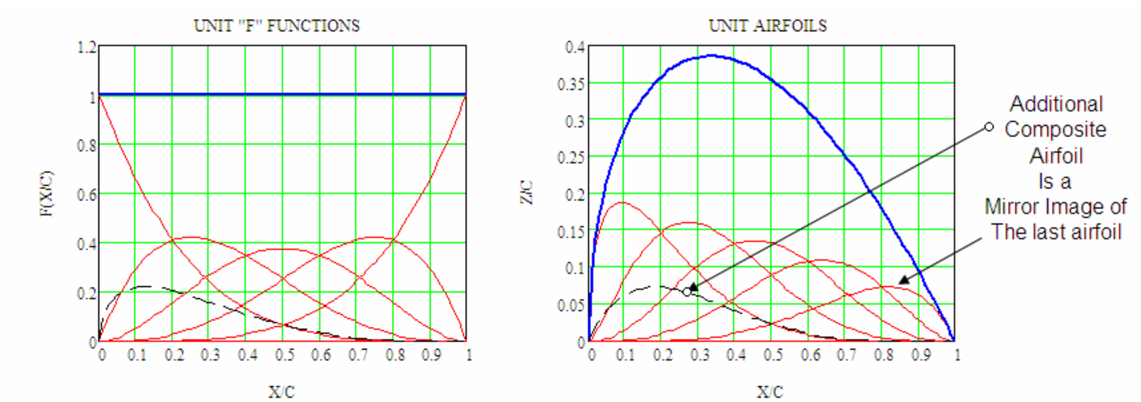

Fig. 2. BPO4 Composite Shape functions and Composite Airfoils With Additional Sharp Nose Composite Airfoil.

The equation for the additional "nose slope" airfoil is: $ZoCs(\psi) = \psi \cdot (1-\psi)^{N+0.5}$ (6)

The corresponding additional shape function term is: $Ss(\psi) = \psi^{0.5} \cdot (1-\psi)^{N-0.5}$ (7)

The shape of an arbitrary airfoil in terms of the composite airfoils including the sharp nose term is given as

$$ZoC(\psi) = \sum_{i=0}^{N} \left(A_i \cdot \zeta_i(\psi) \right) + A_{N+1} \cdot ZoCs(\psi) + \psi \cdot \Delta\zeta_{TE} \tag{8}$$

This equation applies to both the upper and lower surface of a cambered airfoil.

The additional term has only a small effect on airfoils having very little nose camber such as the RAE2822 airfoil as shown by the results of the convergence studies shown in table 1.

	Required Order of Bernstein Polynomial		
	Manufacturing "Exactness"	Measurement "Exactness"	Aerodynamic "Exactness"
Upper Surface	BPO3 (BPO4)	BPO5 (BPO7)	Not Determined
Lower Surface	BPO5 (BPO5)	BPO7 (BPO8)	Not Determined

(without slope term)

Table 1: Effect of the Nose Slope Term on the Convergence of the CST Geometry With a RAE2822 Airfoil Data

The geometry of the RAE2822 as illustrated in figure 2, has matching upper and lower surface radii and zero nose camber.

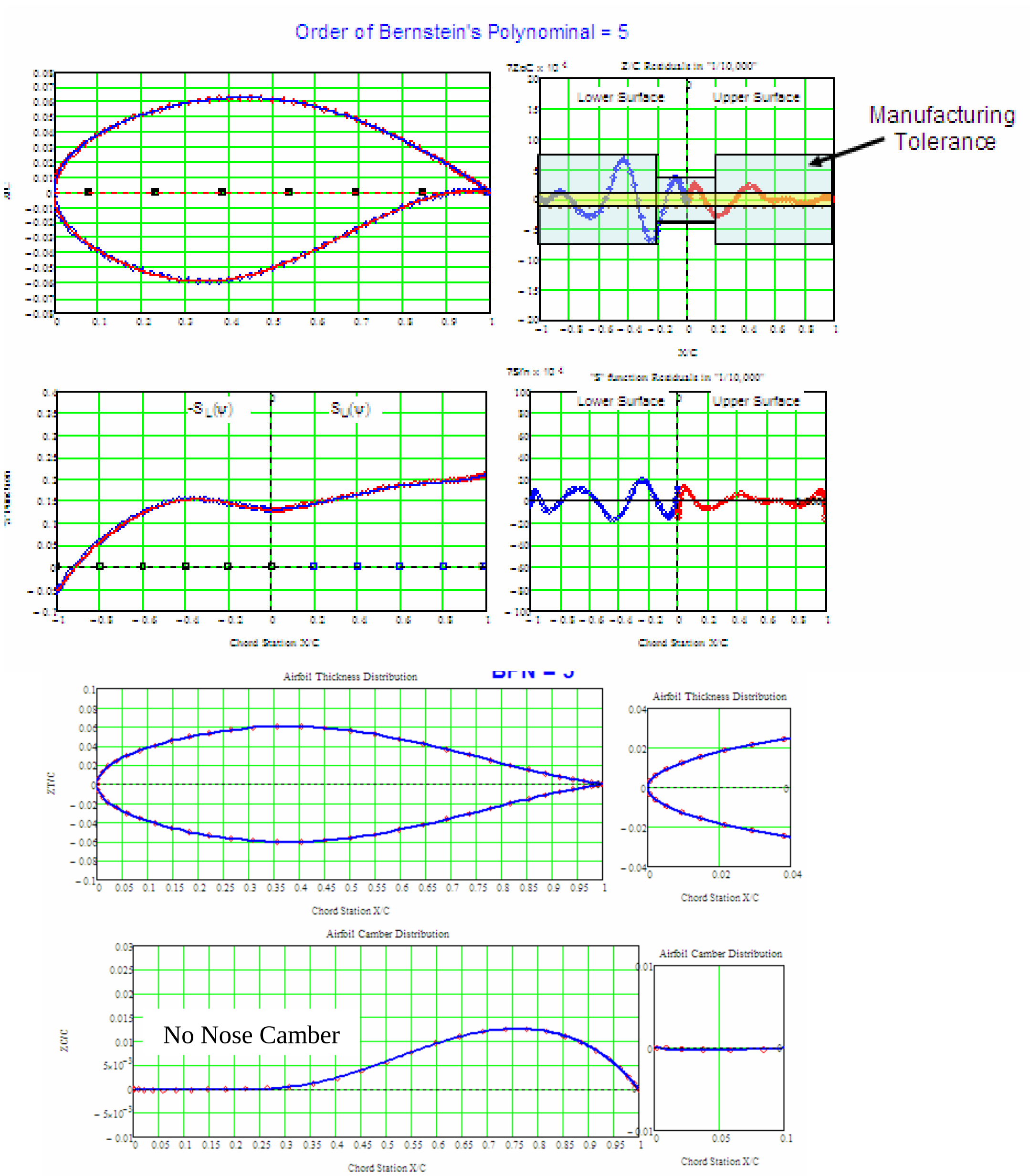

Figure 3: RAE2822 Geometry Characteristics

The NACA6412 airfoil shown in Figures 4 and 5, is an example of an airfoil with significant nose camber and also having differing upper and lower surface leading edge radii.

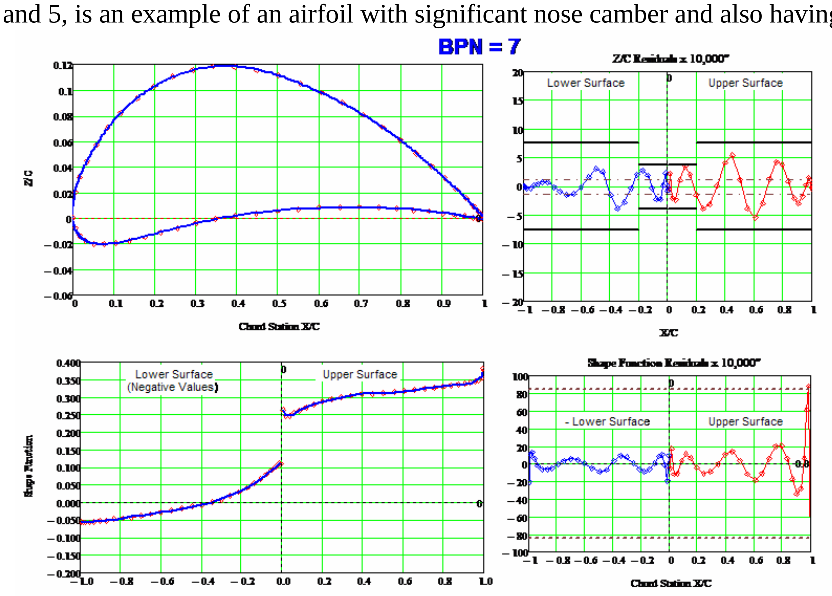

Figure 4. NACA 6412 Airfoil and Shape Function

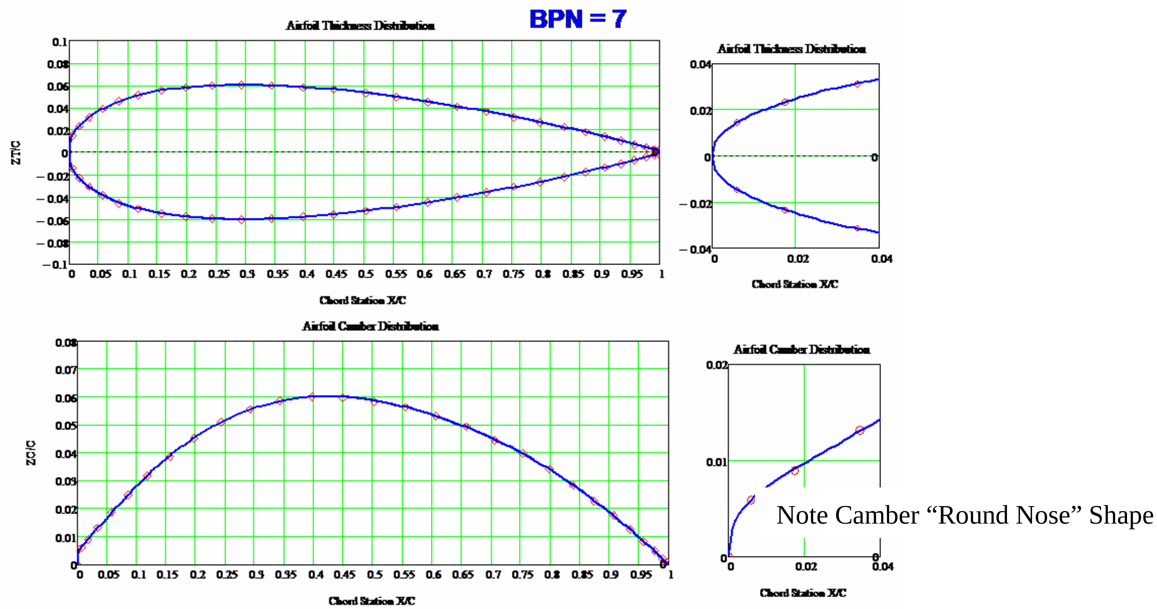

Fig. 5. NACA6412 Camber and thickness distribution

The effect of the differing upper and lower surface radii is evident in figure 5 in the round nose of the camber line.
As shown in table 2, the addition of the sharp nose term reduced to number of terms to match the actual geometry. This also suggests that the addition of the sharp nose term could allow the use of a reduced number of terms for using the CST methodology for design optimization

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	Required Order of Bernstein Polynomial		
	Manufacturing “Exactness”	Measurement “Exactness”	Aerodynamic “Exactness”
Upper Surface	BPO7 (BPO13)	BPO9 (~ BPO15)	Not Determined
Lower Surface	BPO4 (BPO8)	BPO8 (BPO14)	Not Determined

Study Summary (without slope term)

Table 2: Effect of the Nose Slope Term on the Convergence of the CST Geometry With a NACA6412 Airfoil Data

Figure 6 shows an example of a round nose symmetric airfoil having an additional finite nose slope term.

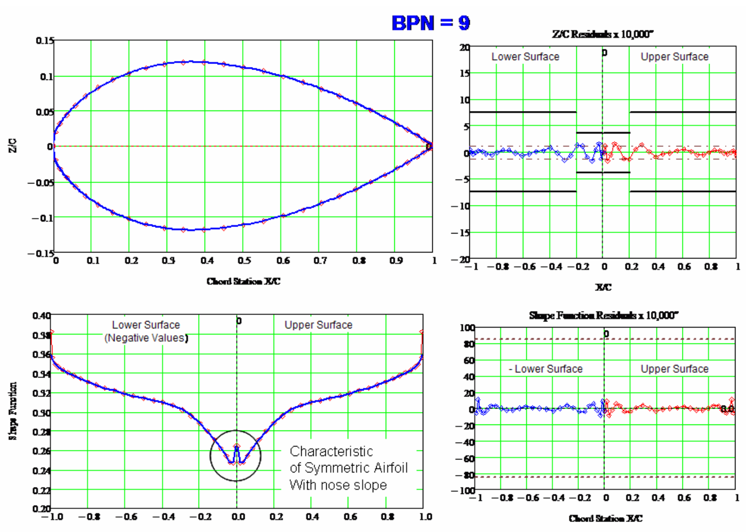

Fig. 6. Symmetric Round Nose Airfoil with additional Finite Nose Slope term

The “hook” in the shape function near the airfoil nose is indicative of the presence of a finite nose slope term.

Application to Camber and Thickness Distributions

The conventional definitions of the thickness coordinates, ζ_T, and the camber coordinates, ζ_C, in terms of the upper surface, ζ_U, and lower surface, z_L, coordinates are respectively:

$$\zeta_T(\psi) = \frac{\zeta_U(\psi) - \zeta_L(\psi)}{2} \quad \text{and} \quad \zeta_C(\psi) = \frac{\zeta_U(\psi) + \zeta_L(\psi)}{2}.$$

The same shape function formulation with the sharp nose modification can be used for describing the upper surface, the lower surface, the thickness distribution and the camber distributions.

Added Benefit for supersonic wing design

The CST sharp edge modification has added benefits for supersonic wing design where the airfoil nose shape is round for a subsonic leading edge design and the airfoil nose shape is sharp for a supersonic leading edge design as shown in figures 7 and 8 respectively.

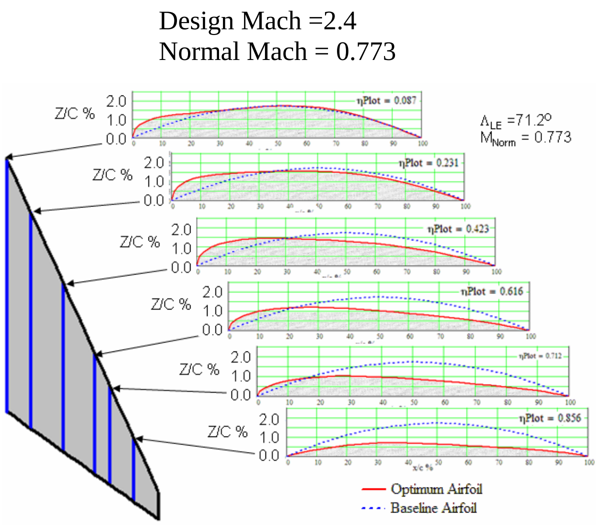

Fig. 7. Subsonic Leading Edge Supersonic Wing Design

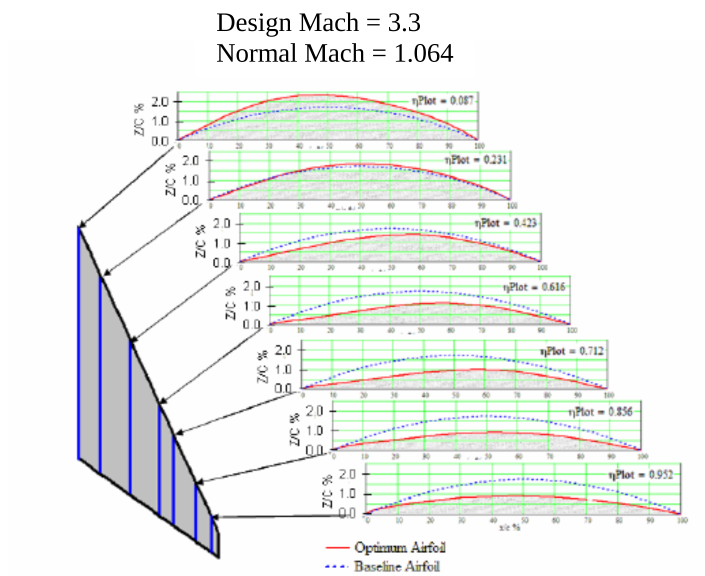

Fig. 8. Supersonic Leading Edge Supersonic Wing Design

Both of these designs can be developed exactly from the round nose class function when the modified shape function formulation with the additional sharp leading edge term is included.

Similarly, supersonic wing having both subsonic and supersonic leading edges such as the cranked leading edge in figure 9 and the ogee wing planform in figure 10 can be designed with a common class function and the modified shape function across the entire planform.

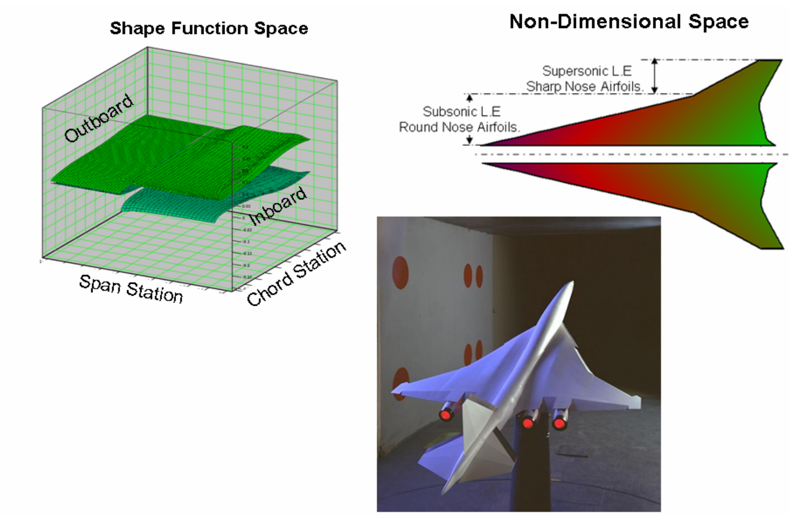

Fig. 9. Cranked Leading Edge Supersonic Wing

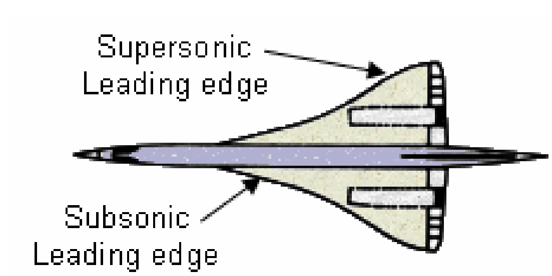

Fig. 10. Supersonic Ogee Wing Planform

Recommendation:

Include the Additional nose slope term for all general CST airfoil and wing definition and /or design optimization studies that include the round nose / sharp trailing edge class function..

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References

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2. Kulfan, B. M., "A Universal Parametric Geometry Representation Method – CST”, AIAA--2007-0062, Jan 2007
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