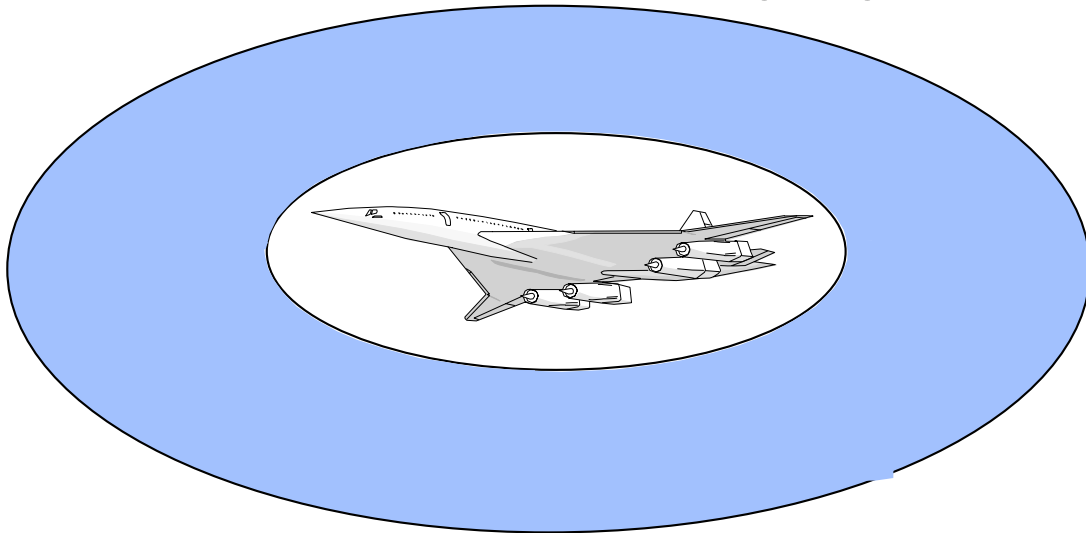


# **Historic Background on Flat Plate Turbulent Flow Skin Friction and Boundary Layer Growth**



## **HSR Airframe Technical Review**

**Los Angeles, California  
February 9 - February 13, 1998**

**Robert M. Kulfan  
Boeing Commercial Airplane Group**

# Topics

- Early Skin Friction Compressibility Prediction Attempts
- Skin Friction Prediction Validation Studies
- Prediction of Flat Plate Turbulent Layer Growth
- Summary / Conclusions

Recent CFD validation studies have shown significant variations in viscous drag predictions between the various methods used by the NASA and industry HSCT organizations. The methods include Navier Stokes CFD codes in which the viscous forces are part of the solutions, and predictions obtained from the different fully turbulent flow flat plate skin friction drag equations used by the various organizations.

The initial objective of this study was to provide an experimental database of fully turbulent flow skin friction measurements on flat plate adiabatic surfaces at subsonic through supersonic Mach numbers and for a wide range of Reynolds numbers. The database could then be used as the initial step in resolving the differences in the viscous predictions.

This database,(Ref 1), was originally assembled in 1960 from selected experiments conducted prior to that time period. The criteria used to select the appropriate test data are described in the reference. Data were also found on turbulent boundary layer velocity profiles and it was therefore possible to analyze other boundary layer properties such as shape factor, displacement thickness and boundary layer thickness.

The data presented in this note was scanned from the figures in the report and then digitized using a highly accurate PC screen digitizer. The digitized data will be released in a report early in 1998.

In the process of extracting the data, statistical analyses were made between the test data and the corresponding predictions of various fully turbulent flat plate skin friction prediction methods. An improved method of predicting compressible turbulent skin friction drag was developed.

Boundary layer profile data measurements are also included along with a new method for predicting boundary layer growth characteristics. These include approximate velocity profile representation, boundary displacement thickness, and boundary layer thickness.

# Why the Interest in Flat Plate Turbulent Boundary layers ?

- First Step in Evaluating Navier Stokes Prediction Methods
- Help Sort Out Appropriate Turbulence Models
- Good Estimate of Viscous Drag of HSCT Type Configurations ( Easy, Quick , Robust and Accurate)
- PD Drag Prediction Methods
- Extrapolation of Wind Tunnel Data to Flight Conditions
- $\delta$  Predictions Used to Size Diverter Height
- $\delta^*$  plus CF Predictions Used to Calculate Spillage and Internal Drag of Flow-Through Nacelles
- Quick Estimate of Surface Temperature
- Provides Physical Insight into Viscous Flow Characteristics

It is felt that the first step in validating the viscous drag predictions of any Navier Stokes code is to make sure that predictions of the local and average skin friction drag and boundary layer must match the “simple” flat plate measured test data over the range of Mach numbers and Reynolds for which the codes will be used. This process will help to evaluate the applicability of the various turbulence models.

Because HSCT configurations have rather thin wings, slender bodies and low cruise lift coefficients, experience has shown that flat plate skin friction calculations provide good estimates of the viscous drag of HSCT type configurations. The predictions are easy, quick, robust and quite accurate.

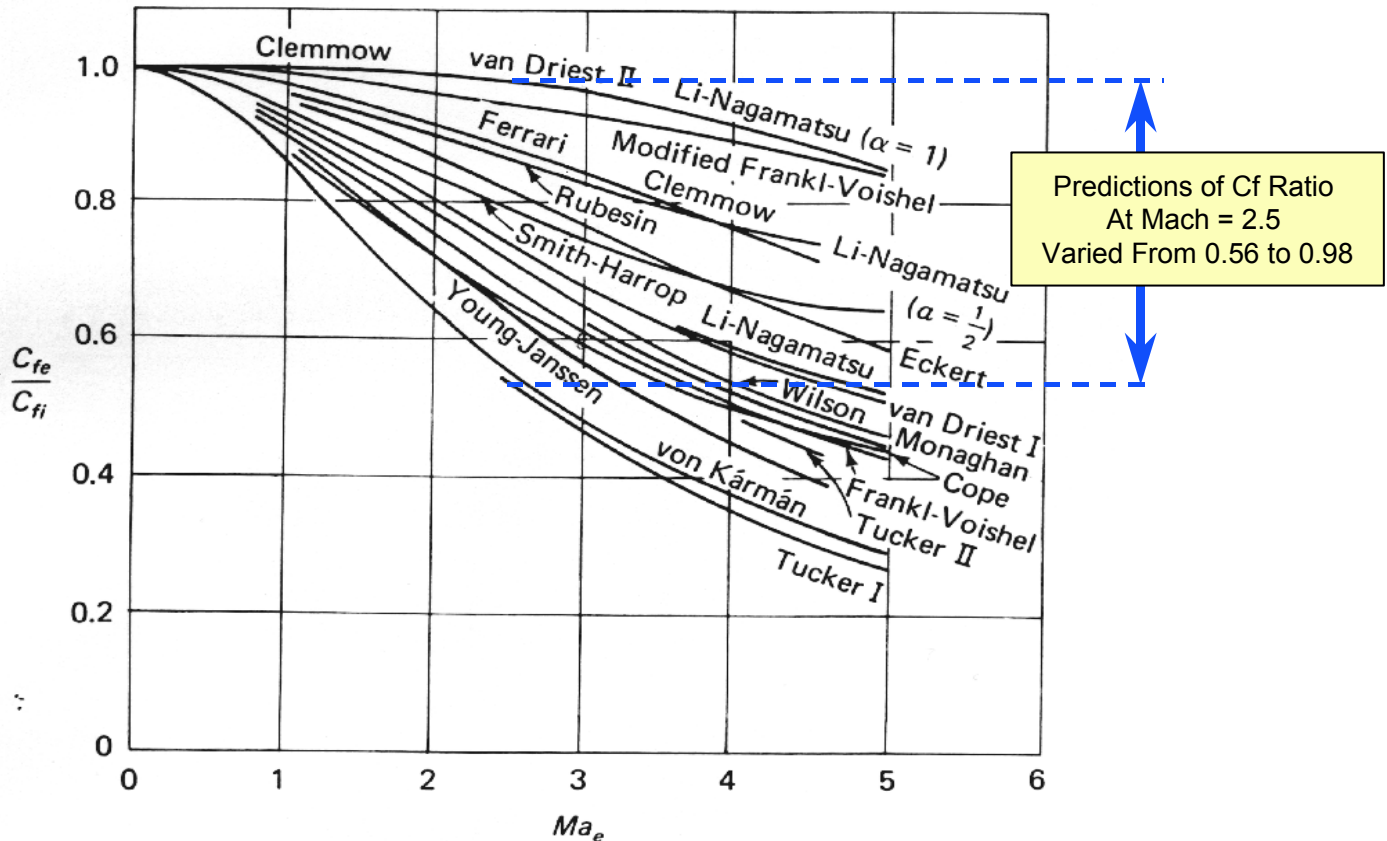
The current PD viscous drag prediction methods are based on flat plate skin friction drag calculations. Currently wind tunnel data is extrapolated to flight conditions using flat plate friction drag predictions.

Flat plate estimates of the boundary layer thickness are used as the preliminary criteria for specifying the boundary layer diverter height for the HSCT nacelle installations. Boundary layer displacement thickness predictions together with CF calculations are used to calculate the spillage and internal drag of wind tunnel flow through nacelles.

Local skin friction calculations corrected for local dynamic pressure effects can be used to estimate local surface temperatures.

The boundary layer thickness information presented in this note also provides some physical insight in to the fundamental features of turbulent flat plate flow.

# Early Predictions of Compressibility Effects on Skin Friction



The initial objective of the study in Reference 1, was to determine the most appropriate semi-empirical method for accounting for compressibility effects on flat plate skin friction drag. Chapman and Kester in Reference 5, after a study of approximately 20 theoretical methods being proposed at that time, found at Mach 2.5, calculated values of the ratio of compressible skin friction drag to incompressible skin friction drag, ( $C_f/C_{fi}$ ), varied from 0.98 to 0.56 depending upon which method was used. Whereas measured values for this ratio varied from about 0.67 to 0.65 over a Reynolds range of  $5.5 \times 10^6$  to  $8.5 \times 10^7$ . The different predictions are shown in the above figure from Reference 2.

In order to establish the validity of the skin friction coefficient relation; an extensive survey was made in Reference 1, to gather reliable experimental data from many independent sources. A rigid set of criteria was adopted as a means of selecting data for a systematic study. This was done to insure that the test conditions closely approximate the theoretical model, and that both the measurement and reduction techniques were such as to yield accurate information.

The most significant of these requirements were:

1. Use only of data obtained by direct force measurements. Reference 9, 16, 25 and 26 discuss the relative merits of various skin friction measurement techniques. The general conclusion is that the most accurate data are obtained by direct force measurements.
2. The flow over the experimental model was to be properly tripped to satisfy the condition of fully turbulent flow.
3. Measurements were to be made at stations far enough downstream of the trips to allow the flow to reach a "naturally" turbulent character.
4. Experimental results were to be presented in terms of the properly determined effective turbulent length.

## Reference Temperature Approach

- Incompressible Skin Frictions Equations Can be Used to Calculate Compressible Skin Friction if an “Appropriate” Reference Temperature is Used To Calculate  $\rho$  and  $\mu$  in the equations:

eg:  $C_{fi} = 0.295 [\log(\text{Re}_x)]^{-2.45}$  ---- Modified Schultz-Grunow Eqn

$$C_f = \frac{\rho^*}{\rho_\infty} C_{fi}^* \qquad \text{Re}_{x^*} = \text{Re}_x \frac{\rho^* \mu_\infty}{\rho_\infty \mu^*}$$

- Assuming the Static Pressure is Constant Across the Boundary Layer:

$$\frac{\rho^*}{\rho_\infty} = \frac{T_\infty}{T^*}$$

- The Compressible Skin Friction Equation becomes:

$$C_f = 0.295 \frac{T_\infty}{T^*} \left[ \log \left( \text{Re}_x \frac{T_\infty}{T^*} \frac{\mu_\infty}{\mu^*} \right) \right]^{-2.45}$$

It is perhaps worth emphasizing the empirical nature of what is called "theory" in this report and the necessity, therefore, that this theory should be compared with data from more than one source. The basis of the theory selection was; first, it had to agree, of course, with test data within the scatter of that data, and secondly, it had to be based on good physical reasoning. All of the theoretical flat plate formulations involve disposable constants that have been determined empirically. Thus, as is the rule for all empirical formulae, the theory should be, strictly speaking, only be applied where it has been justified by experiment; however, because there is a physical basis to this theory, it is believed that some extrapolation should be permissible. This is equally true for current Navier Stokes CFD codes where viscous flow effects are determined using various turbulence models which approximate the flow phenomena.

Statistical analyses of the differences between the flat plate theory and the test data will be used to establish both the consistency of the test data, and the adequacy of the theoretical predictions. This will allow more effective use of the data for use in CDF viscous drag prediction validation studies.

All of the skin friction theories shown in the previous figure were developed by assuming that compressible turbulent skin friction drag could be obtained using well known incompressible skin friction equations by evaluating all of the fluid properties that appear in the incompressible equations at some appropriate reference temperature,  $T^*$ . This assumption parallels the analytical transformation methods that had been used in laminar boundary compressible flow analyses.

The assumption of an effective reference temperature in essence implies that the turbulent boundary shape and height are not strongly affected by Mach number. This will be further examined in this paper.

# Methods Used To Determine a Reference Temperature

- Similar to Laminar Flow Transformation
- For Adiabatic Wall Conditions The Reference Temperature Equation is of the Form:

$$\frac{T^*}{T_\infty} = 1 + K_r \cdot r(\sigma - 1)M_\infty^2$$

- “Constant”  $K_r$  Determined By:
  - Wall Temperature ( Correction Too Large )
  - Determined Experimentally -- Sommer / Short
  - Correlation of Experimental  $C_f$  data -- Kulfan; Spaulding / Chi; White
  - Velocity Averaged Enthalpy Across the Boundary Layer -- Monaghan
  - Semi-Analytic -- Van Driest

Numerous ideas were for an appropriate reference were proposed by the various researchers. This accounts for the widely differing predictions of compressibility effects on skin friction drag as shown in Figure 4. Some of the early concepts used to define the reference temperature equation coefficients are shown in the figure.

These include:

- Use of the surface temperature ----this provided too large a compressibility correction
- Determined experimentally by specially designed experiments, --- Sommer / Short (Ref 12)
- Determine by correlation of  $C_f$  predictions with test data. --- Spaulding / Chi (Ref 2), White (Ref 2), Kulfan ( shown later in this report).
- Velocity averaged enthalpy across a boundary layer ---- Monaghan (Ref 27)
- Semi-analytic formulations -- Van Driest (Ref 2)

## Current Approach

- Use Existing Local Skin Friction Data to Validate Incompressible Equation
- Selected “Quality” Data from Many Sources
- Statistical Analysis to Assess Scatter of Data and Consistency of Predictions
- Use Selected Reference Temperature Equation(s) to Transform Experimental Values of “Rex” and “Cf” to Equivalent Incompressible Values.
- Statistical Analysis to Assess Scatter of the Experimental Data and the ability of the Reference Temperature to Convert the Measured Friction Data to Equivalent Incompressible Values.
- Apply to Selected Reference Temperature equation(s) to Existing CF Data as Additional Verification

Cf ----- Local Skin Friction Coefficient

CF ----- Average Skin Friction Coefficient

In the current study the reference temperatures selected for evaluation included: the Monaghan mean enthalpy equation, and the Sommer / Short equation. Previous studies have shown both to provide accurate assessments of compressible skin friction. The Sommer / Short method is the current method used in Boeing Seattle PD methods.

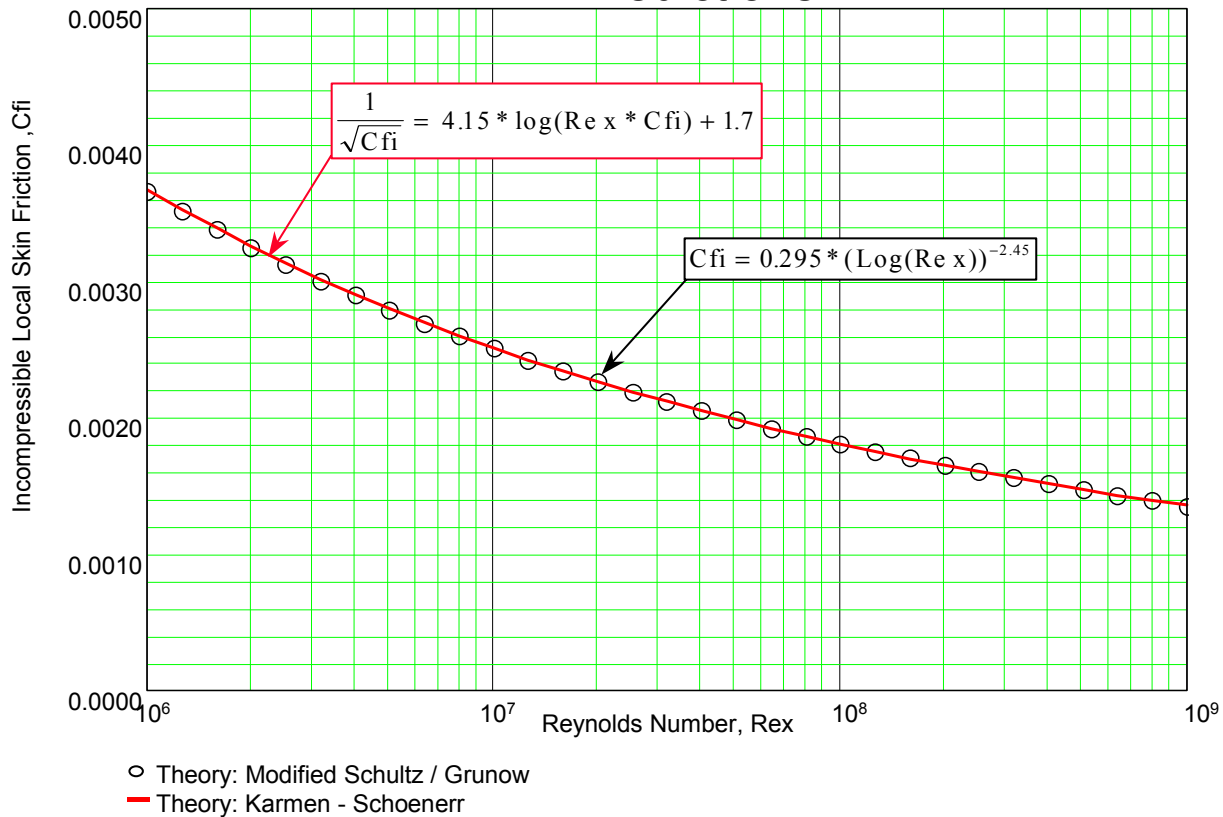
Experimentally, it is much easier to obtain force measurements of local skin friction drag than of average skin friction drag. Consequently, the initial step in the current evaluation process was to compare incompressible local skin friction data with the most generally accepted incompressible skin friction equations. Data from many different sources were used.

The selected reference temperature were then used to transform measured compressible local skin friction data to equivalent incompressible Cf and Reynolds numbers. Statistical analyses of the transformed compressible friction data were compared with the incompressible predictions, to assess the adequacy

of the selected reference temperatures to account for the compressibility effects.

Subsequently, the same process was then applied to available average skin friction data.

# Comparison of Incompressible Local Skin Friction Predictions



Page 8

The most widely accepted in compressible local skin friction equation is the Karmen / Schoenherr equation:

$$\frac{1}{\sqrt{C_{fi}}} = 4.15 \cdot \log(\text{Re}_x \cdot C_{fi}) + 1.7$$

This is compared in this figure with the less sophisticated modified Shultz / Grunow equation.

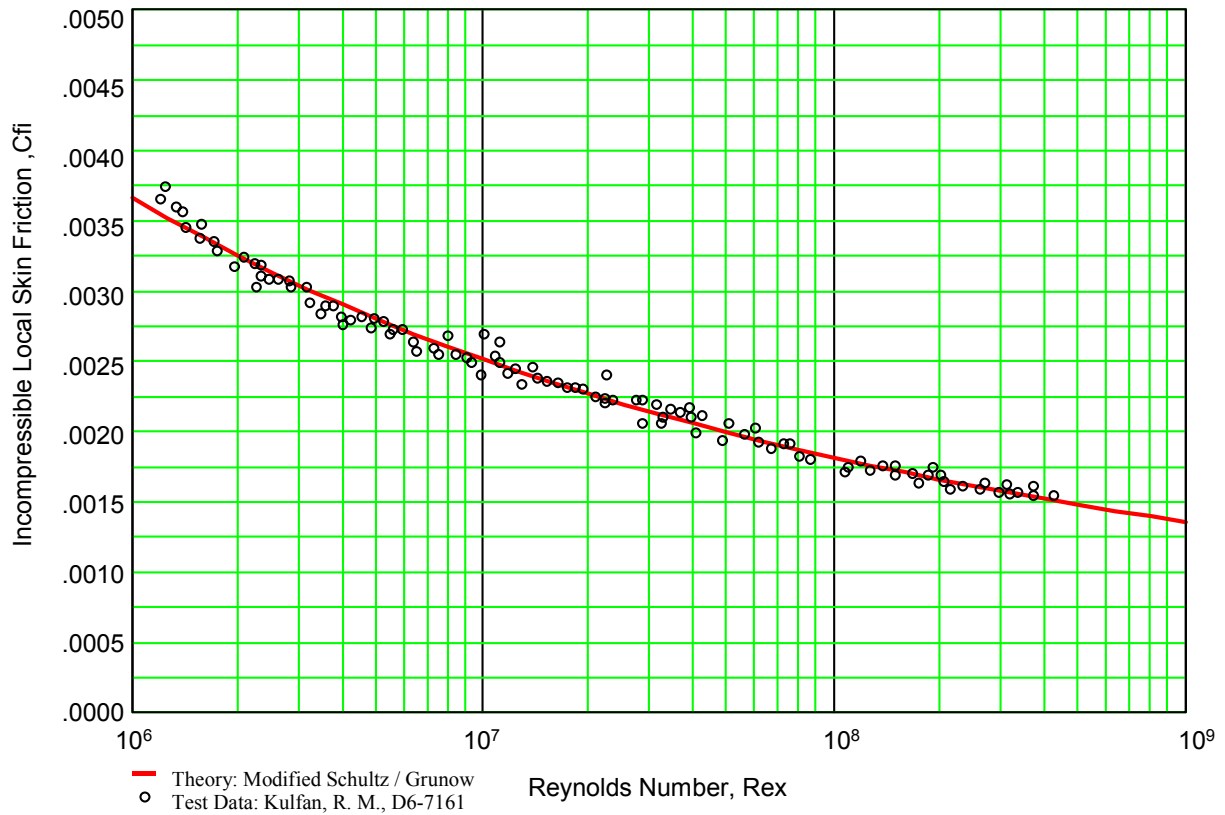
$$C_{fi} = 0.295 \cdot (\text{Log}(\text{Re}_x))^{-2.45}$$

The modification was simply replacing the standard constant “0.288” by “0.295”.

The “mean” difference between the  $C_f$  values calculated by the Karmen-Schoenherr equation and by the modified Shultz-Grunow equation was -0.0000031 over the complete Reynolds number range. The standard deviation was calculated to be 0.00000452. Consequently, the simpler Shultz / Grunow equation was used in the current study.

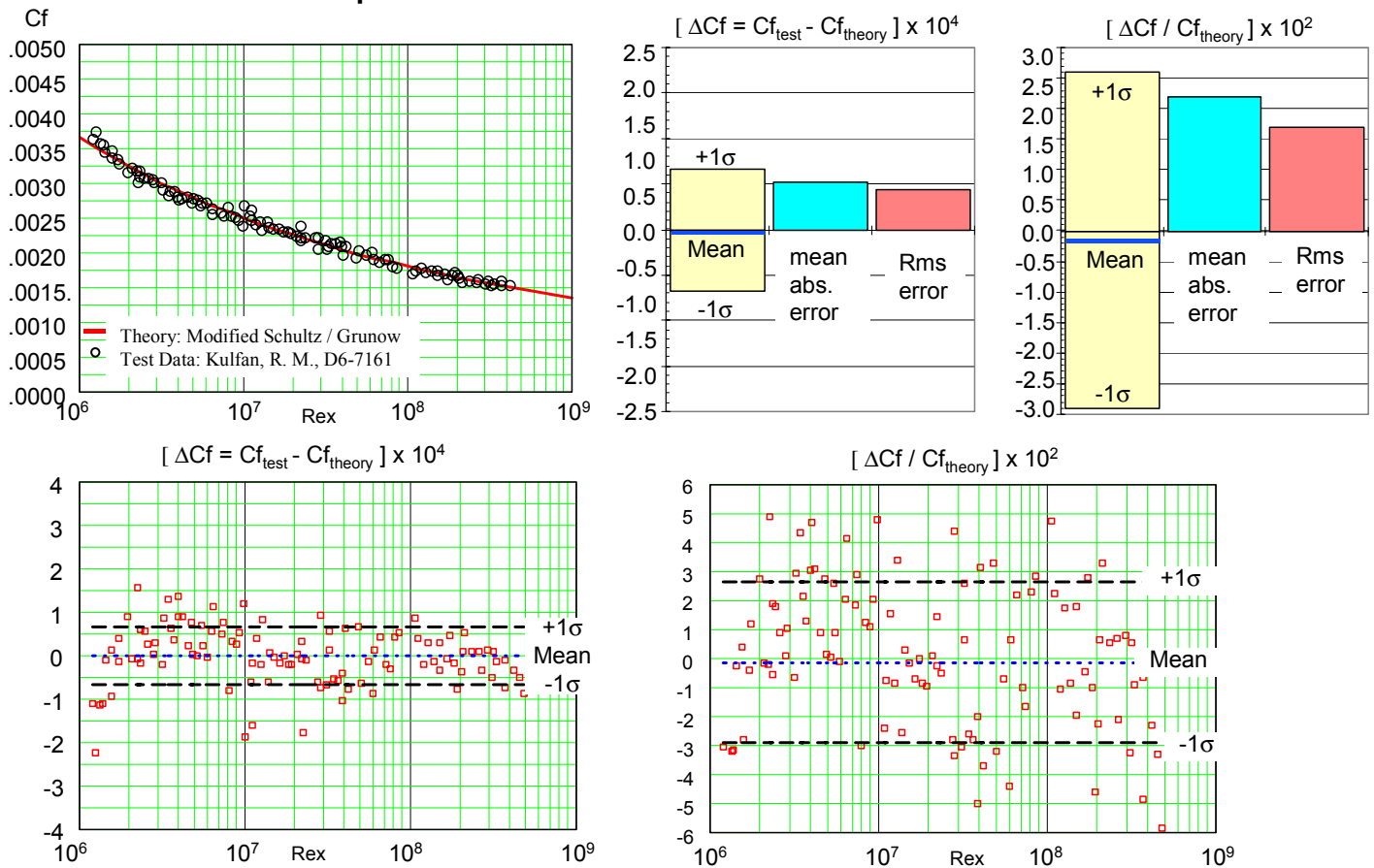


# Incompressible Local Skin Friction Data



This figure compares measured incompressible local skin friction data from References 15, 25 and 39. The test data appears to scatter about the theoretical predictions for the entire Reynolds number range of the test data.

# Incompressible Local Skin Friction Data



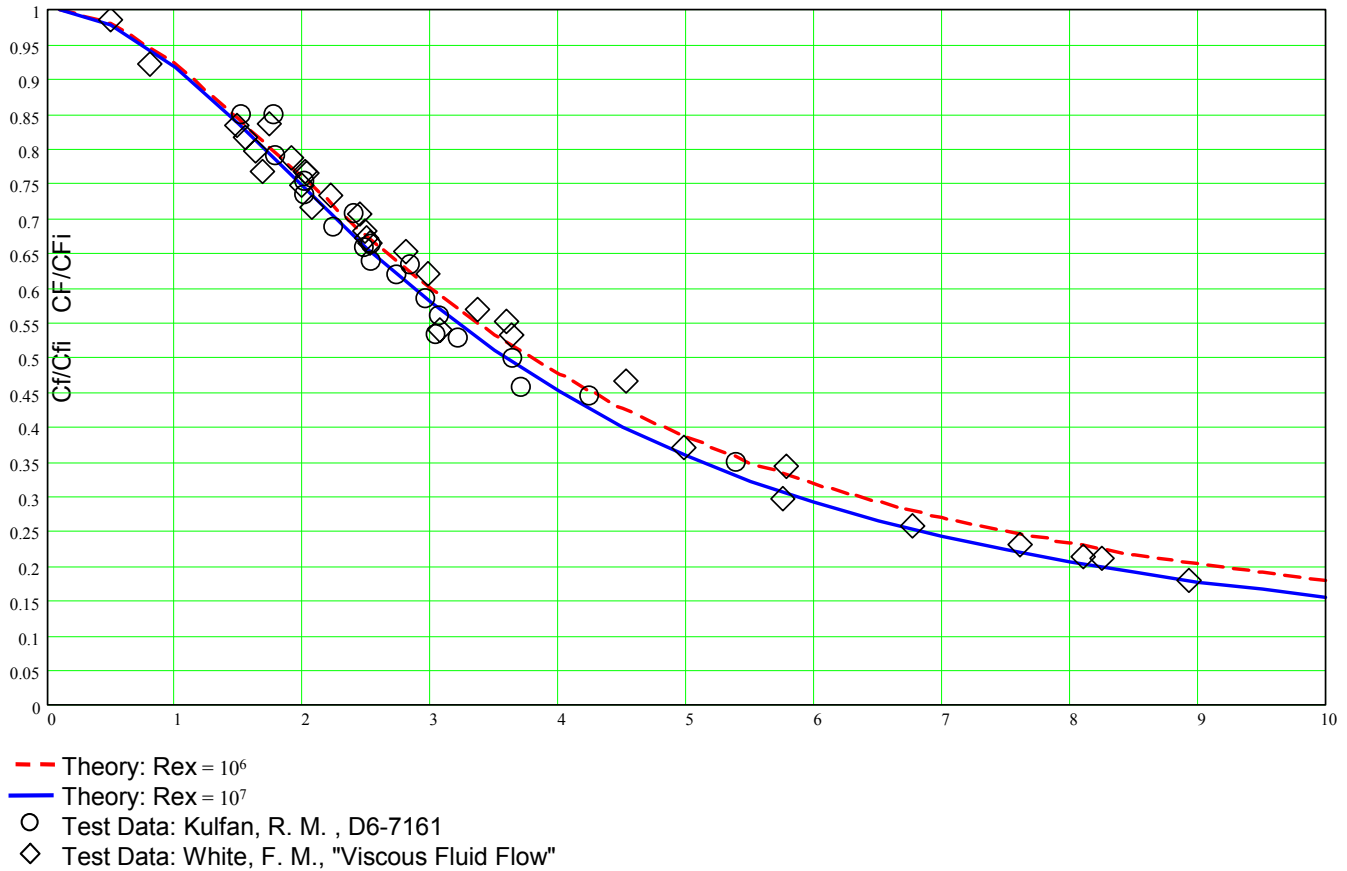
Statistical analysis of the differences between the test data and corresponding  $C_f$  predictions shows that the mean of the differences is  $\Delta C_f = -.000000671$  which corresponds to an average difference of 0.13%. The standard deviation of data about the mean is approximately 0.7 counts of drag ( $\Delta C_f = 0.000067$ ) which corresponds to 2.8% of the corresponding predicted value.

The constant 0.288 in the original Shultz / Grunow equation would result in a mean difference between the test and theory of - 2.6%

The modified Shultz / Grunow equation therefore appears to provide an accurate estimate of incompressible local skin friction coefficient over the entire range of Reynolds Numbers covered by the test data.

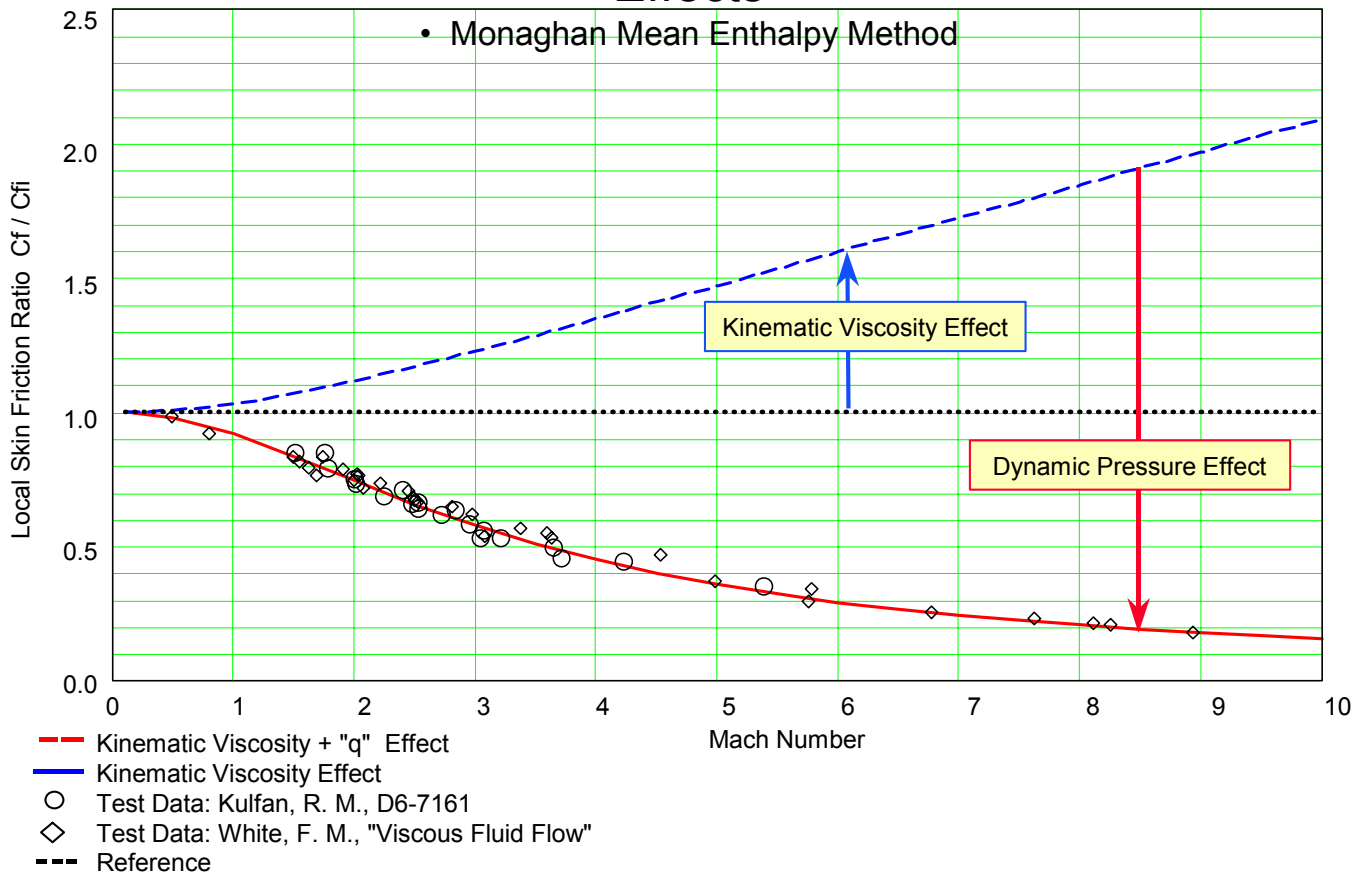
# Prediction of Compressibility Effects on Skin Friction

• Monaghan Mean Enthalpy Method



This figure includes comparisons of the predicted effects of Mach number on the ratio of compressible skin friction to incompressible skin friction at the same Reynolds. The experimental data are from thirteen independent experiments. The sources of the test data are given in References 1 and 2. The test data correspond to Reynolds number between  $10^6$  and  $10^7$ . The theoretical predictions shown in the figure were obtained using the Monaghan  $T^*$  equation. The predictions appear to match the Mach number trends quite well.

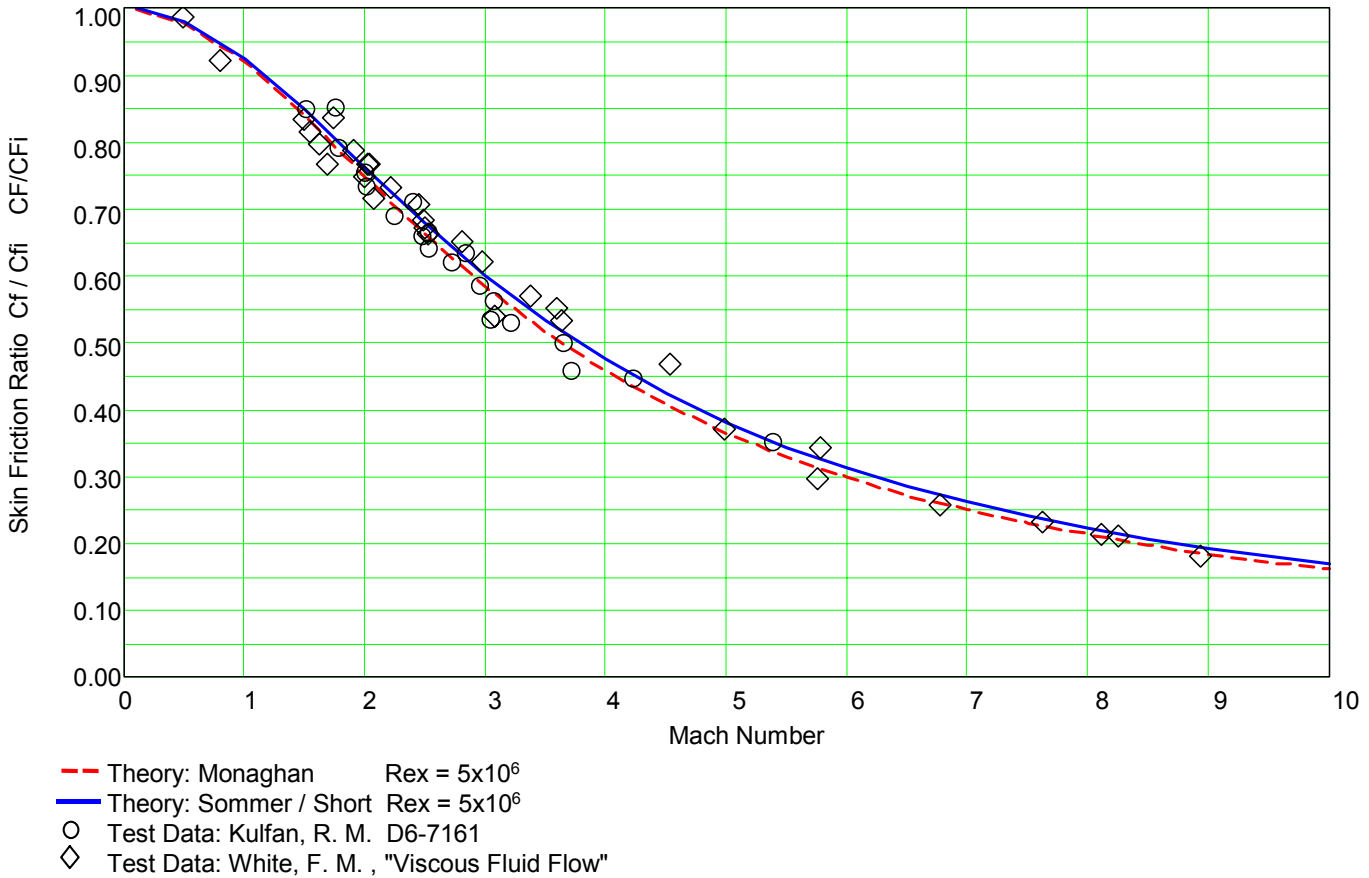
# Compressibility Component Effects



This figure shows the elements of the compressibility corrections. The reference temperature reduces the kinematic viscosity and therefore decreases the effective Reynolds number. This increases the effective skin friction coefficient. The dynamic pressure effect associated with reduction in effective density overpowers the kinematic viscosity and results in the reduced skin friction coefficient when corrected back to free stream reference conditions.

# Comparison of Compressibility Effects Predictions

Skin Friction Ratio  $C_f / C_{fi}$   $CF/CF_i$

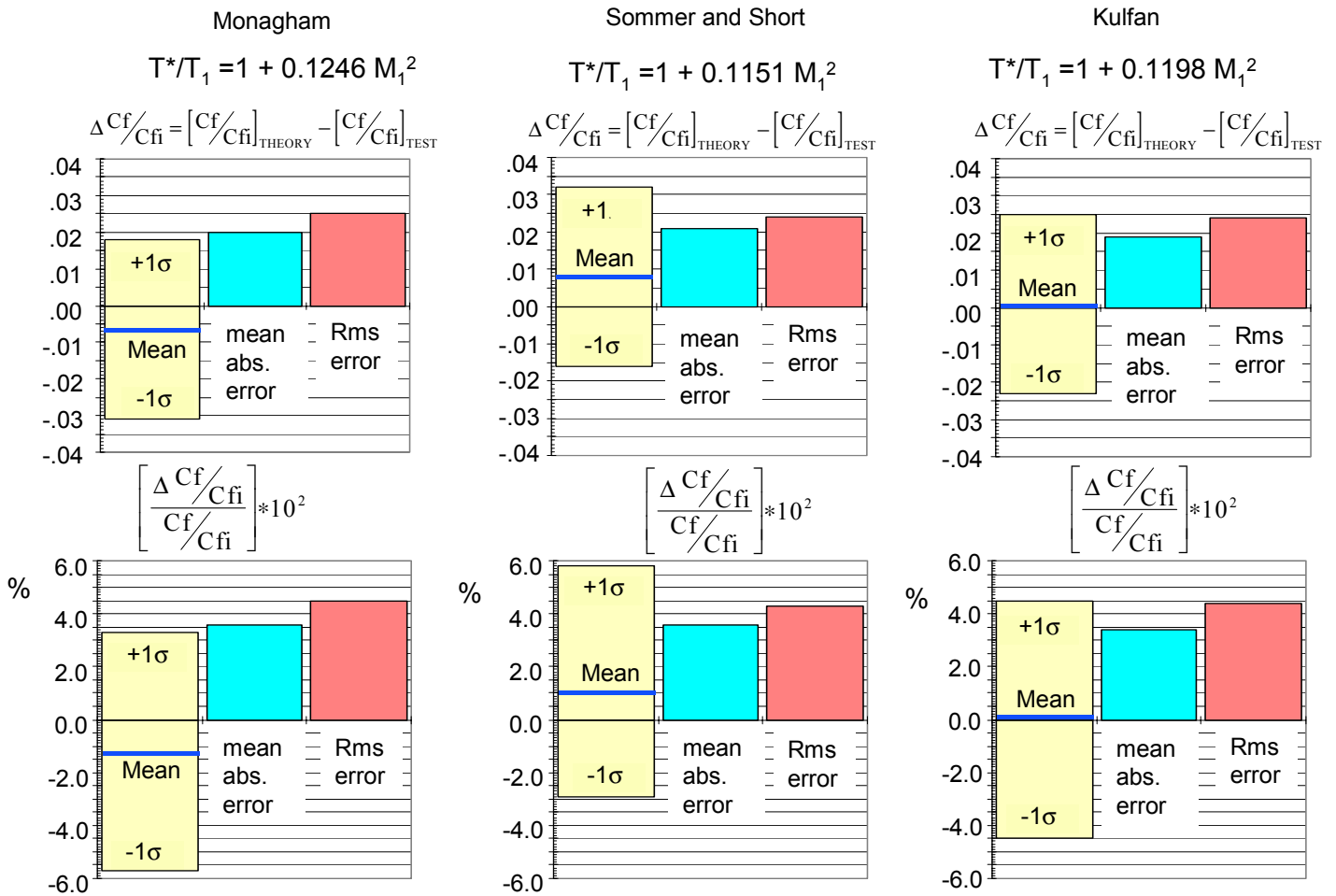


This compares the compressible skin friction predictions obtained using the Monaghan  $T^*$  equation with the predictions obtained using the Sommer-Short  $T^*$  equation.

The Sommer-Short  $T^*$  equation results in compressible skin friction values consistently higher than predicted using the Monaghan method. It was for this reason that the Boeing US SST program switched from the Monaghan method to the Sommer-Short method.

The full scale SST performance predictions were obtained from wind tunnel data corrected to full scale conditions. Wind tunnel skin friction drag is higher than the full scale conditions. Using higher skin friction values calculated by the Sommer -Short method resulted larger skin friction corrections. This resulted in higher L/D assessments for the SST.

# Evaluation of Reference Temperature Equations



Statistical analyses of the differences between  $C_f$  predictions and the corresponding test data are shown in this figure. The theoretical predictions were obtained using three different  $T^*$  equations. The “scatter” in the test - theory increments are essentially equal. The mean of the differences between the test and theory, however differs between the predictions obtained using the different  $T^*$  equations.

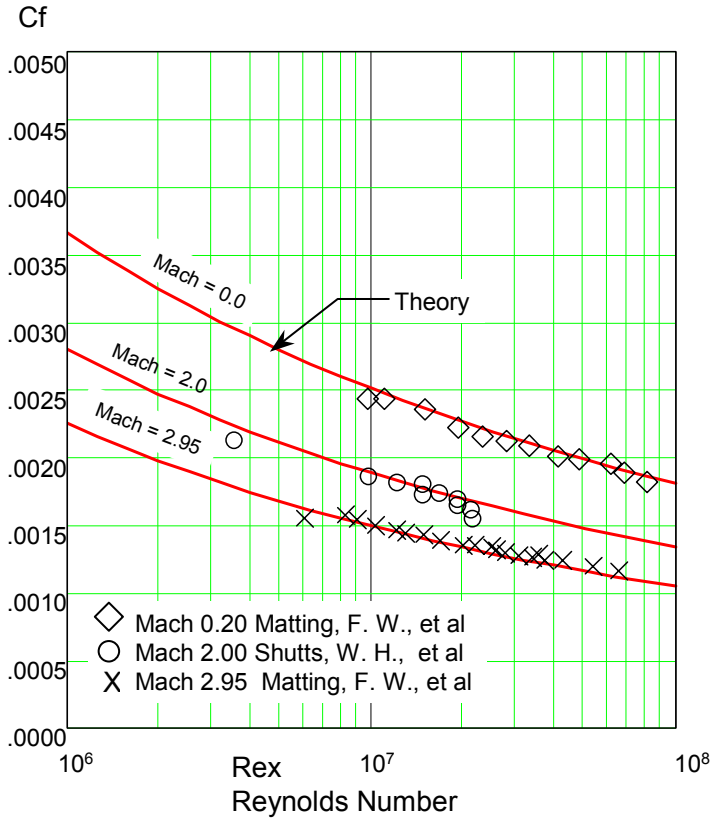
The “mean” of the theory - test differences obtained using the Monaghan  $T^*$  equation is approximately 1% low. The “mean” of the theory - test differences obtained using the Sommer-Short  $T^*$  equation is approximately 1% high. The constant for the Kulfan  $T^*$  equation was therefor chosen to be the average of the Sommer-Short and the Monaghan constants.

This essentially resulted in a mean error between the test data and the theoretical predictions of zero.

The test data scatter about the mean has a standard deviation of about 4.5%. This large scatter is in part due to the variations of Reynolds number of the test data. The Reynolds number for the test data  $10^6$  to  $10^7$ .

# Conversion of Compressible Cf Data to Equivalent Incompressible (Cfi)<sub>eq</sub> Data

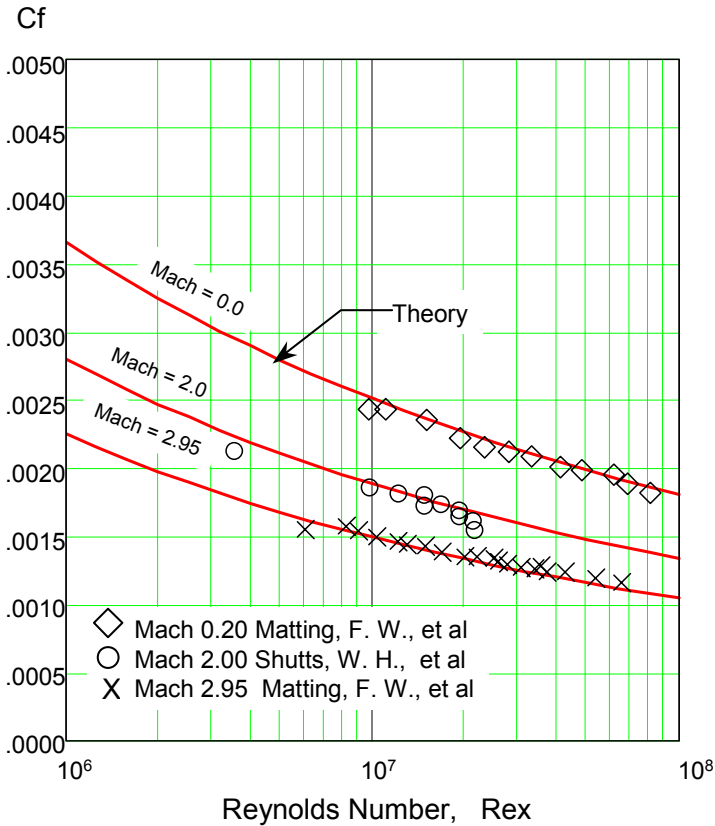
- Kulfan T\* Equation
- Modified Schultz-Grunow Cf Equation



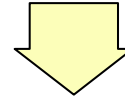
This figure contains comparisons of theoretical predictions of  $C_f$  with test data for three Mach numbers from 0.0 to approximately 3.0. The theory in this figure used the Kulfan T\* equation.

# Conversion of Compressible Cf Data to Equivalent Incompressible (Cfi)<sub>eq</sub> Data

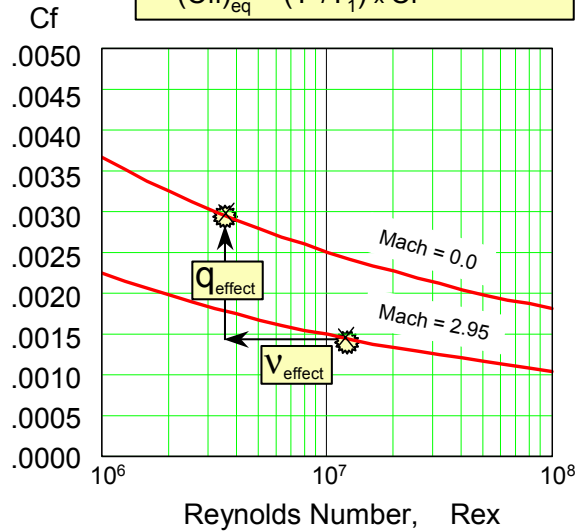
- Kulfan T\* Method
- Modified Schultz-Grunow Cf Equation



$$Cf = 0.295 \cdot \frac{T_1}{T^*} \left[ \log \left( Rex \cdot \frac{\mu}{\mu^*} \cdot \frac{T_1}{T^*} \right) \right]^{-2.45}$$



- $(Rex)_{eq} = (Rex) \times (T_1/T^* \mu_1/\mu^*)$
- $(Cfi)_{eq} = (T^*/T_1) \times Cf$



In order to assess the accuracy of the Cf predictions to account for compressibility or Mach number effects, the test data were converted to equivalent incompressible values of Cfi and Reynolds number.

An example of this transformation is shown in the Figure.

The Equivalent incompressible Reynolds number is less than the actual test Reynolds number. The equivalent incompressible skin friction coefficient is higher than the actual measured skin friction coefficient.

The Reynolds number correction has been calculated using the Sutherland viscosity equation. The resulting equation is:

$$\frac{T_1}{T^*} \frac{\mu_1}{\mu^*} = \frac{1}{\left( \frac{T^*}{T_1} \right)^{2.5}} \frac{198.7 + T^*}{198.7 + T_1}$$

The temperature in the above equation is in degrees Rankine. For the current study the local static temperature was assumed to be 70 deg F. This corresponds to 529.6 deg R.

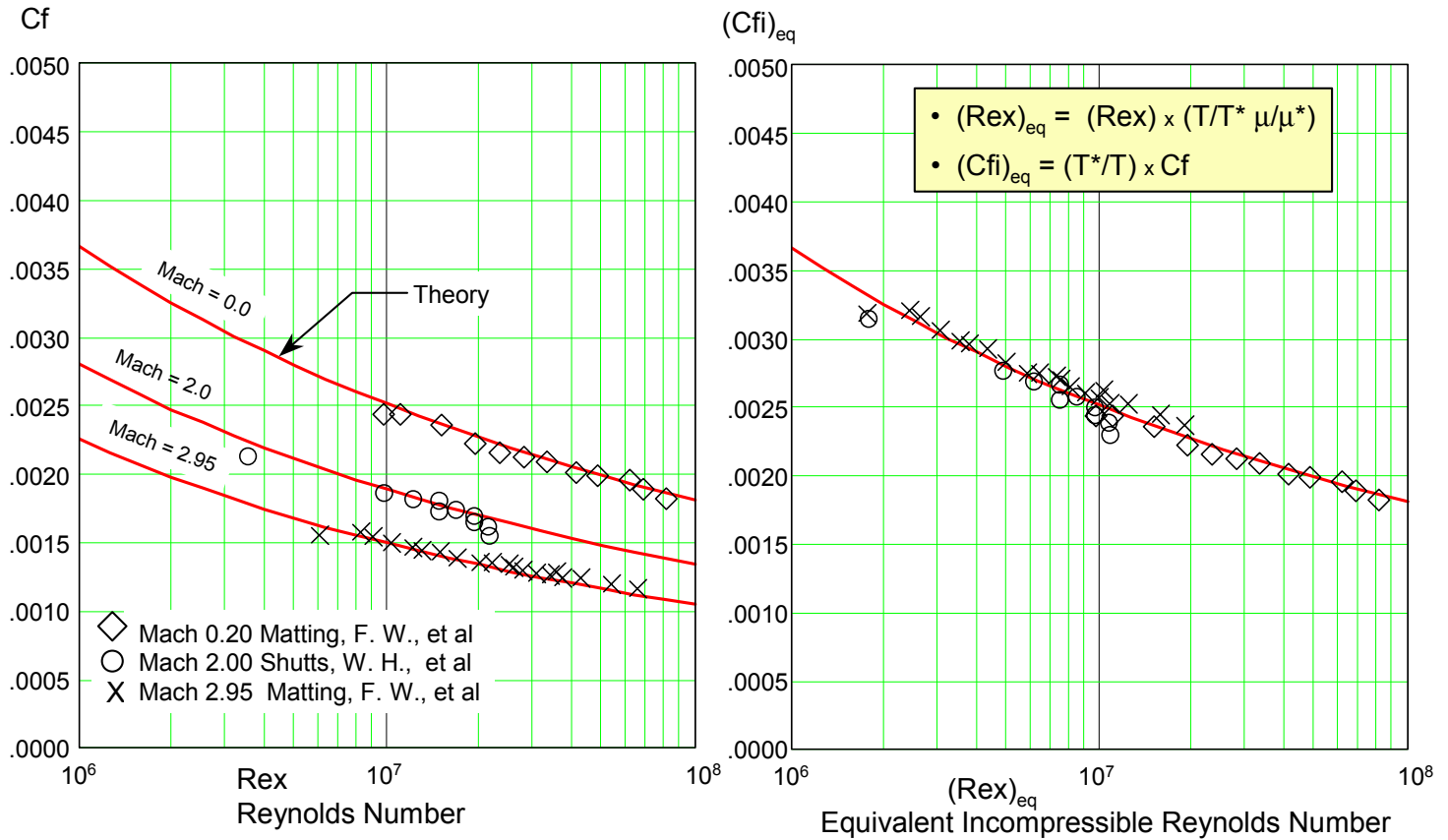
$$T_1 = 459.6 + 70$$

$$T^* = T_1 \left( \frac{T^*}{T_1} \right)$$



# Conversion of Compressible Cf Data to Equivalent Incompressible $(Cf)_{eq}$ Data

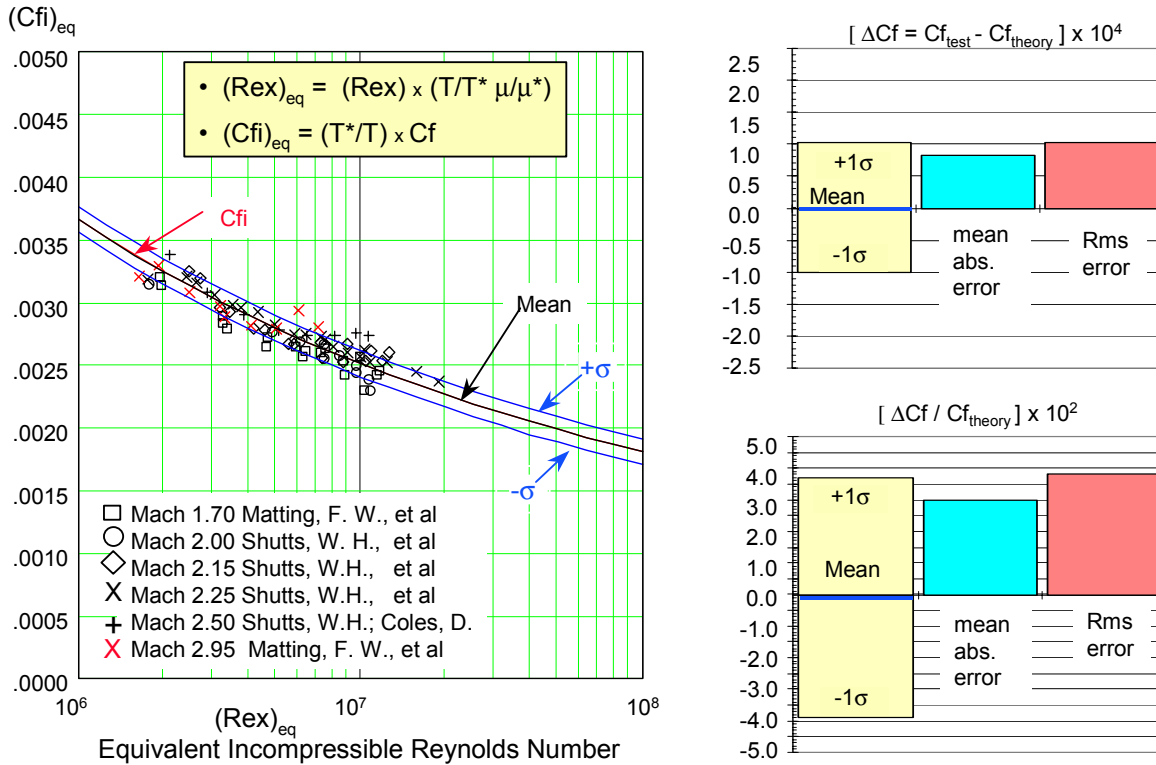
- Kulfan  $T^*$  Method
- Modified Schultz-Grunow Cf Equation



This transformation procedure, as shown in the Figure, “collapses” all of the test data about the incompressible skin friction curve. This approach can provide a convenient means to assess the accuracy of the theoretical methods to account for compressibility effects simultaneously over a range of Mach numbers and Reynolds numbers.

# Conversion of Compressible Cf Data to Equivalent Incompressible (Cfi)<sub>eq</sub> Data

- Kulfan T\* Method
- Modified Shultz-Grunow Cf Equation



This shows transformed experimental data for six different sets of test data obtained at Mach numbers from 1.7 to 2.95. The incompressible Mach number data from the previous plot has not been included in the above figure since it is desired to assess the ability of the different T\* equations to account for Mach number effects on skin friction.

The figure includes the statistically determined differences between the transformed equivalent incompressible skin friction and the modified Shultz-Grunow theoretical Cf predictions. The Kulfan T\* equation was used for the transformation process. The “mean” of the differences between the transformed skin friction data and the incompressible Cf predictions is essentially.

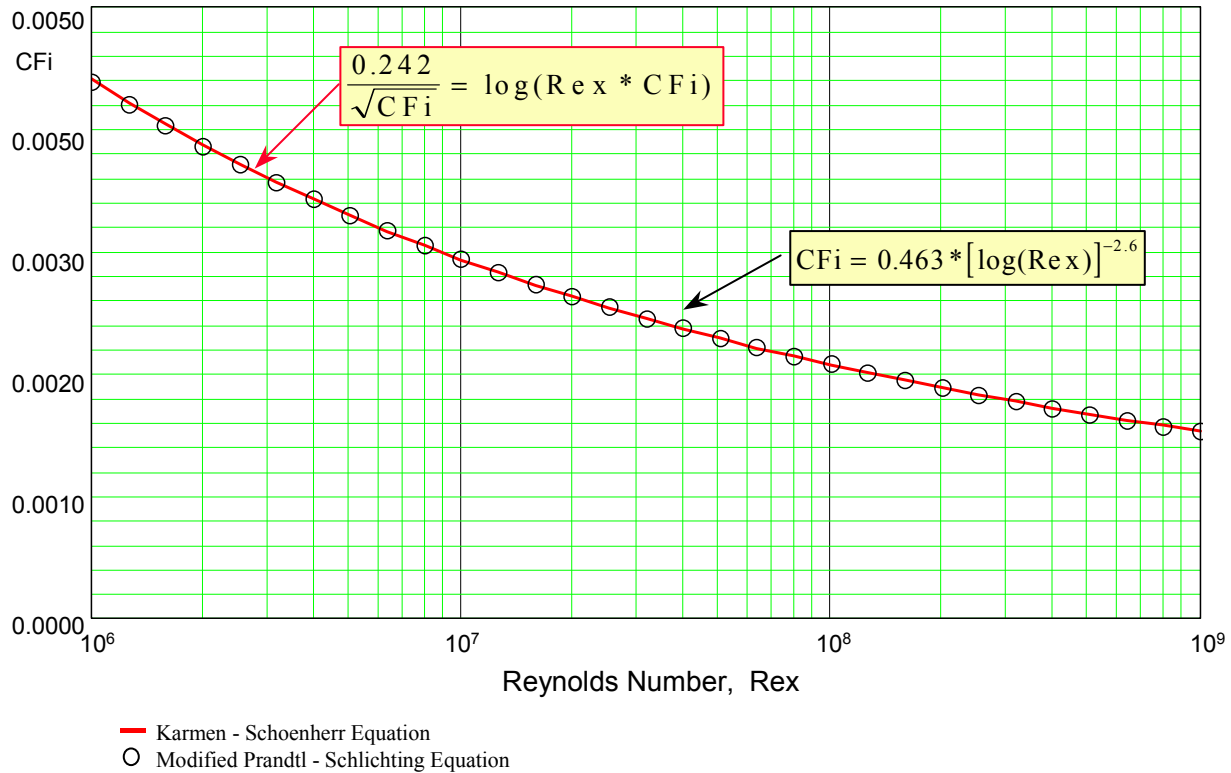
The “scatter” of the test has a standard deviation of about 1 drag count ( $\Delta Cf \sim 0.0001$ ). This corresponds to about a 3.8% scatter of the test data about the theoretical Cf predictions over the entire Reynolds number range and Mach number conditions represented by the test data.

The table below shows the results obtained different T\* equations. On the average, the Monaghan predictions tend to underestimate the test data by about 0.3 counts or 1.2% and the Sommer-Short predictions are about 0.3 counts high corresponding to about 1.0%. The Kulfan T\* method provides the best estimate of the compressibility effects.

	Monaghan T* Eqn.		Sommer-Short T* Eqn		Kulfan T* Eqn.	
	$\Delta Cf$ (counts)	$\Delta Cf / Cf$ (%)	$\Delta Cf$ (counts)	$\Delta Cf / Cf$ (%)	$\Delta Cf$ (counts)	$\Delta Cf / Cf$ (%)
Mean	-0.301	-1.2	.308	1.00	.00071	.085
$\sigma$	1.022	3.8	.985	3.0	1.066	4.0

The “scatter” in the compressible theoretical - experimental transformed skin friction increments are only slightly higher than the scatter in the incompressible data shown in figure 10. ( 0.7 counts versus 1 count). This is most likely because the transformation to equivalent Cf amplifies the magnitude of the Cf values and hence the absolute differences.

## Comparison of Incompressible Average Skin Friction Methods



The most widely accepted in compressible local skin friction equation is the Karmen / Schoenherr equation:

$$\frac{0.242}{\sqrt{C_{fi}}} = \log(Re_x \cdot C_{fi})$$

This is compared in this figure with the less sophisticated modified Prandtl / Schlichting equation.

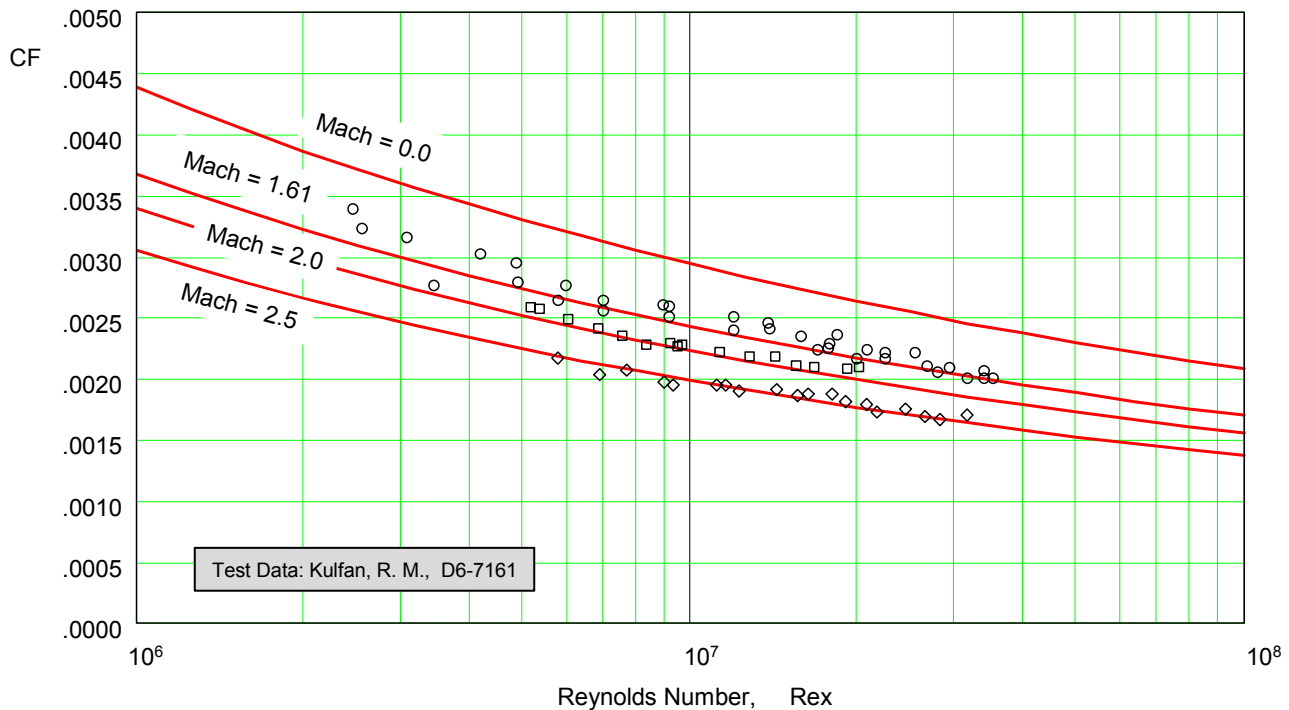
$$C_{fi} = 0.463 (\log Re_x)^{-2.6}$$

The modification was simply replacing the standard constant “0.460” by “0.463”. The mean difference between the  $C_{fi}$  values calculated by the Prandtl-Schlichting equation and by the Karmen-Schoenherr equation was -0.0000013 over the complete Reynolds number range. The standard deviation was calculated to be 0.00000678. Consequently, the simpler modified Prandtl-Schlichting equation was used in the current study.

It is interesting that though out their technical careers, Prandtl and Von Karmen often tackled the same fluid dynamic problem. Their results almost always differed in the analytical formulations and the form of the equations describing the flow phenomena. Computed results were always within a few percent of each other.

# Average Skin Friction Coefficient: Test vs Theory

- Kulfan T\* Method
- Modified Prandtl-Schlichting CF Equation



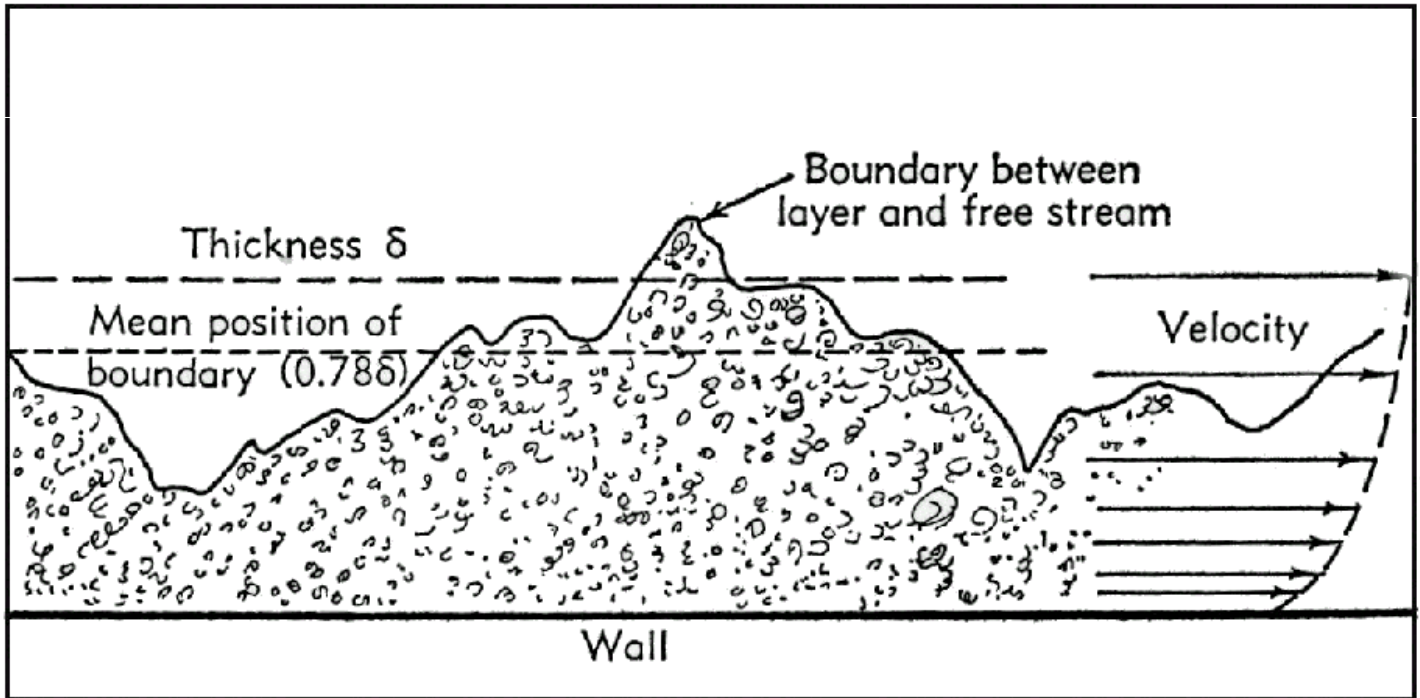
Comparisons between theoretical and experimental average skin friction data are shown in the figure. The lack of additional test data is attributed to the difficulty in obtaining average skin friction data by direct force measurements. Most often, average skin friction data are obtained by application of the momentum integral equation to boundary layer velocity profile measurements. The uncertainties of the interference between the pitot probes used for the measurements and the surface introduces errors that are difficult to correct.

The data shown in the Figure for Mach 2.0 and Mach 2.5, were obtained from force measurements on the cylindrical portion of a cone-cylindrical body of revolution. The Mach 1.61 data were obtained with an ogive - cylinder body of revolution. Three dimensional effects are considered to be small on the cylindrical sections. However determining the “effective origin” for the flow over the cylindrical can certainly introduce substantial errors.

The theoretical predictions match the Mach 2.0 and Mach 2.5 data quite well. Theory underestimated the friction drag at Mach 1.6. This is believed to be due to a bias in the test data.

The results of the data correlations shown in this paper indicate that comparisons with local skin friction data is the best approach to evaluate any methods for prediction of flat plate skin friction drag.

# Turbulent Boundary Layer



The Edge of a Turbulent Boundary Layer is Sharp But Very Irregular

In the current HSCT studies estimates of the boundary layer height are used to specify the height of the boundary layer diverter to keep the inlet from ingesting portions of the boundary layer.

During the course of the investigation described in Reference 1, experimental measurements of velocity profiles were found. It was also then possible to study the growth characteristics of a turbulent boundary layer over a flat plate. A method was developed to predict the growth of a turbulent boundary layer on a flat plate. This method has been revised in the current study.

The edge of a turbulent boundary layer bounded by a free stream of negligible turbulence has a sharp but very irregular outer limit as shown above. The velocity tends to approach the free stream velocity asymptotically. Hence the definition of the thickness of a turbulent boundary layer is subject to many variations. A common definition of the edge of the boundary layer,  $\delta$ , is the height at which the velocity is equal to some percentage of the free stream value. Typically a value of 0.995 is used.

## Parameters Used to Characterize the Growth of a Boundary Layer

- Boundary Layer Thickness,  $\delta$

- Displacement Thickness, 
$$\delta^* = \int_0^{\infty} \left[ 1 - \frac{\rho}{\rho_{\infty}} \frac{u}{U_{\infty}} \right] dy$$

- Momentum Thickness, 
$$\theta = \int_0^{\infty} \frac{\rho}{\rho_{\infty}} \frac{u}{U_{\infty}} \left[ 1 - \frac{u}{U_{\infty}} \right] dy$$

- Shape Factor, 
$$H = \frac{\delta^*}{\theta}$$

Because of the asymptotic nature of a turbulent boundary layer, other parameters are often used to characterize the boundary layer growth. These include the displacement thickness,  $\delta^*$ , the momentum thickness,  $\theta$ , and the shape factor  $H$ .

The displacement thickness defines the amount that the flow streamlines diverge around the surface because of the boundary. Calculations of the displacement thickness are used in the estimation of the spillage characteristics and the internal drag of flow-through nacelles on wind tunnel models.

The momentum thickness on a flat plate is directly related to the average skin friction coefficient as:

$$\theta = (X CF)/2$$

One technique used to determine average skin friction on a flat plate to measure the velocity profile, integrate the experimental velocity profile to obtain  $q$ . Then the average skin friction coefficient is calculated using the above equation.

The shape factor,  $H$ , is often used to predict the separation tendency of a boundary layer with an adverse pressure gradient.

# Calculation Of Boundary Layer Characteristics

1. Approximate Velocity Profile as:  $\frac{u}{U_{\infty}} = \left(\frac{y}{\delta}\right)^{\frac{1}{N}}$

2. Determine “N” Experimentally

3. Calculate  $\delta^*$  From :  $\delta^* = \theta^* H_i^* \left(\frac{H}{H_i}\right)$

$$\theta = \frac{X \cdot CF}{2}$$

4. Calculate  $\delta$  From :  $\frac{\delta^*}{\delta} = \int_0^1 \left[ 1 - \frac{\rho}{\rho_{\infty}} \left(\frac{y}{\delta}\right)^{\frac{1}{N}} \right] dy$  and  $\delta = \frac{\delta^*}{\left(\frac{\delta^*}{\delta}\right)}$

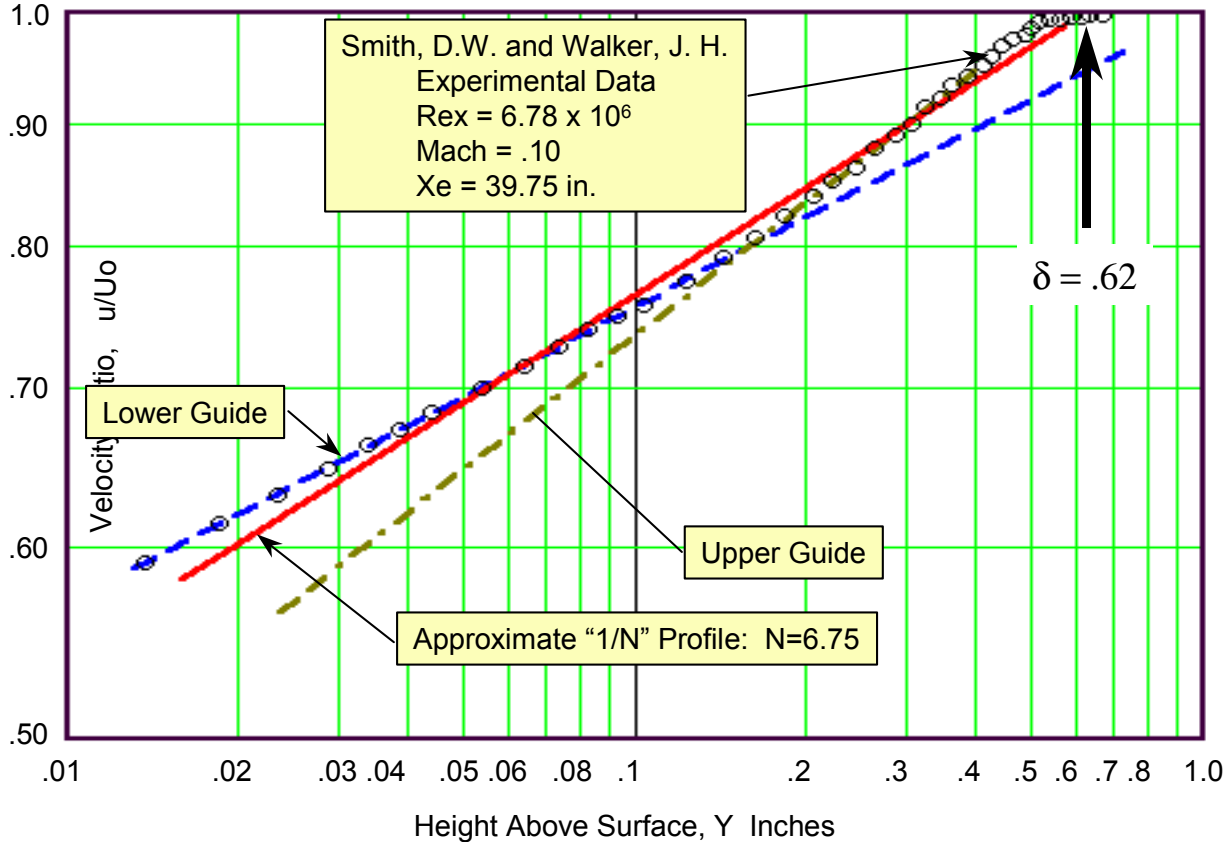
Often in boundary layer studies, it is convenient to represent the velocity profile by a power law relation of the form:

$$\frac{u}{U_{\infty}} = \left(\frac{y}{\delta}\right)^{\frac{1}{N}}$$

This approximate form of the turbulent boundary velocity profile has been used to develop a four step process for predicting the boundary layer thickness. The boundary layer thickness is defined as the height at which the velocity is essentially equal to the free stream velocity.

The elements incorporated in the process for calculating the boundary layer thickness are summarized above, and will be discussed in greater detail in subsequent figures.

# Experimental and Approximate Velocity Profile



The disposable constant for the empirical equation has been determined from correlations of a large number of measured velocity profiles from six independent sources.

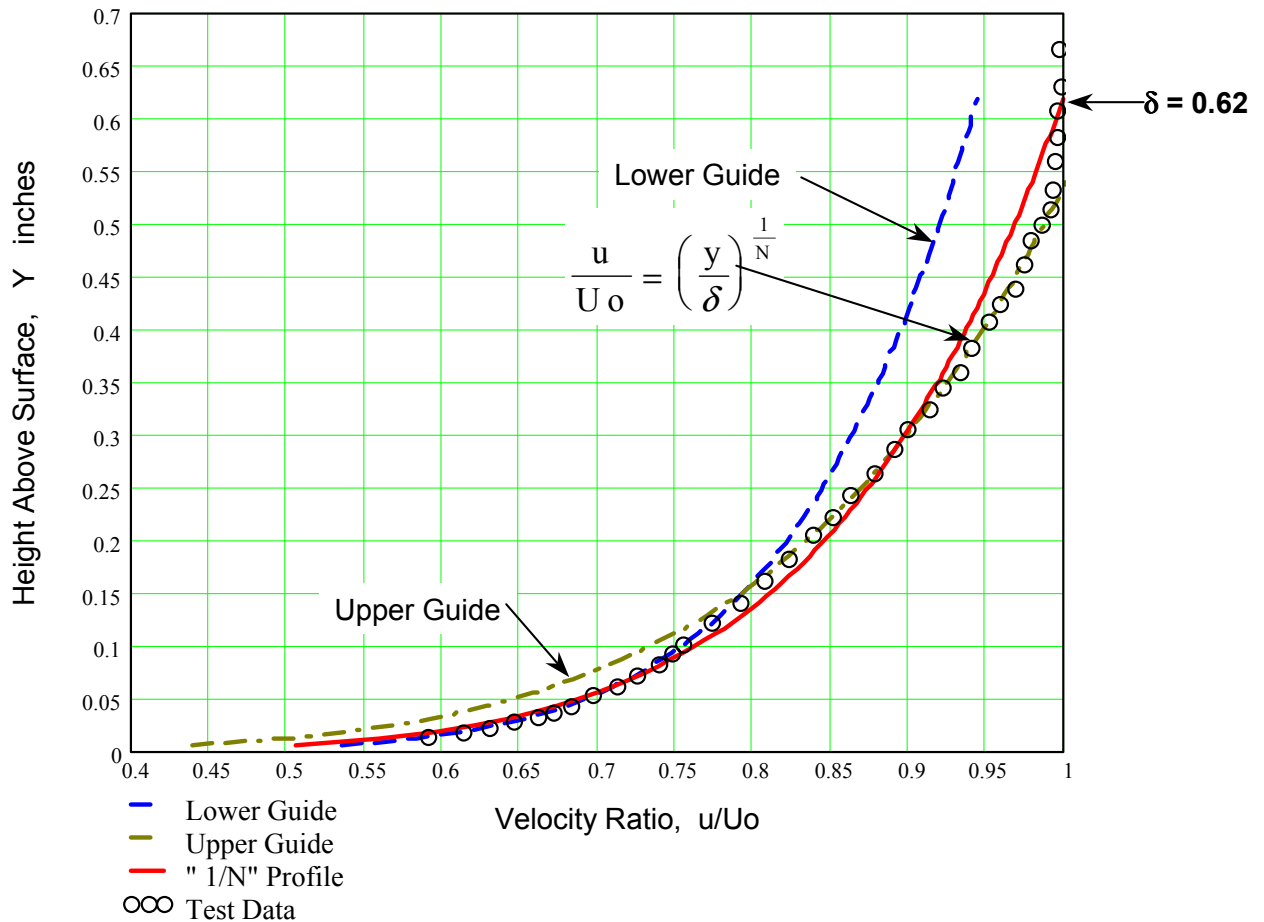
This figure is a typical logarithmic plot of experimental profile measurements and shows the approximate velocity profile representation. Plots such as this indicate that the region on the boundary layer near the surface, and the upper portion can each be represented by a distinct straight lines. This is indeed as it should be, since a more accurate description of a turbulent boundary layer requires the use of two functions.

These include the "law of the Wall" which applies near the surface and the "Law of the Wake" which applies to the intermediate/outer portion of the boundary layer, (References 16,19,23 28,29). These straight lines were used to systematically select a mean line representation of the entire boundary layer.

The velocity profile exponent "N" is determined by the slope of the mean line. The corresponding value of the boundary layer thickness,  $\delta$ , is defined as the height where the mean line intersects the value of  $u/U_o = 1.0$



# Experimental and Approximate Velocity Profiles



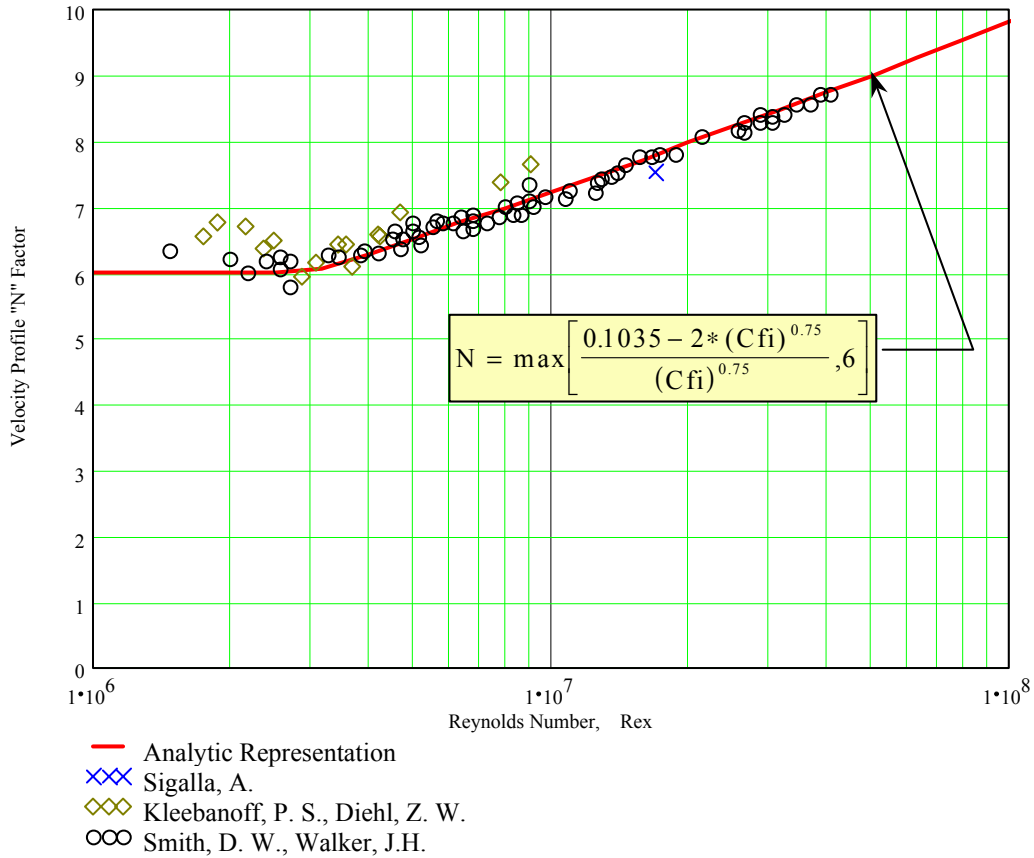
This is a conventional plot of the velocity profile data shown in the previous chart. The various boundary layer growth characteristics were calculated from the measured velocity profile data and also using the approximate "power law" velocity profile. The results are summarized in the table below.

	Measured Profile	Approximate Profile	"Error" %
$\delta^*$	0.0803	0.0801	-0.25
$\theta$	0.0592	0.0613	3.5
H	1.357	1.307	-3.7

The approximate velocity profile "matched" to fit the experimental velocity profile by the process described on the previous page, does provides a good approximation to the turbulent boundary layer growth characteristics.

# Incompressible Velocity Profile “N” Factor

$$u/U_o = (y/\delta)^{1/N}$$

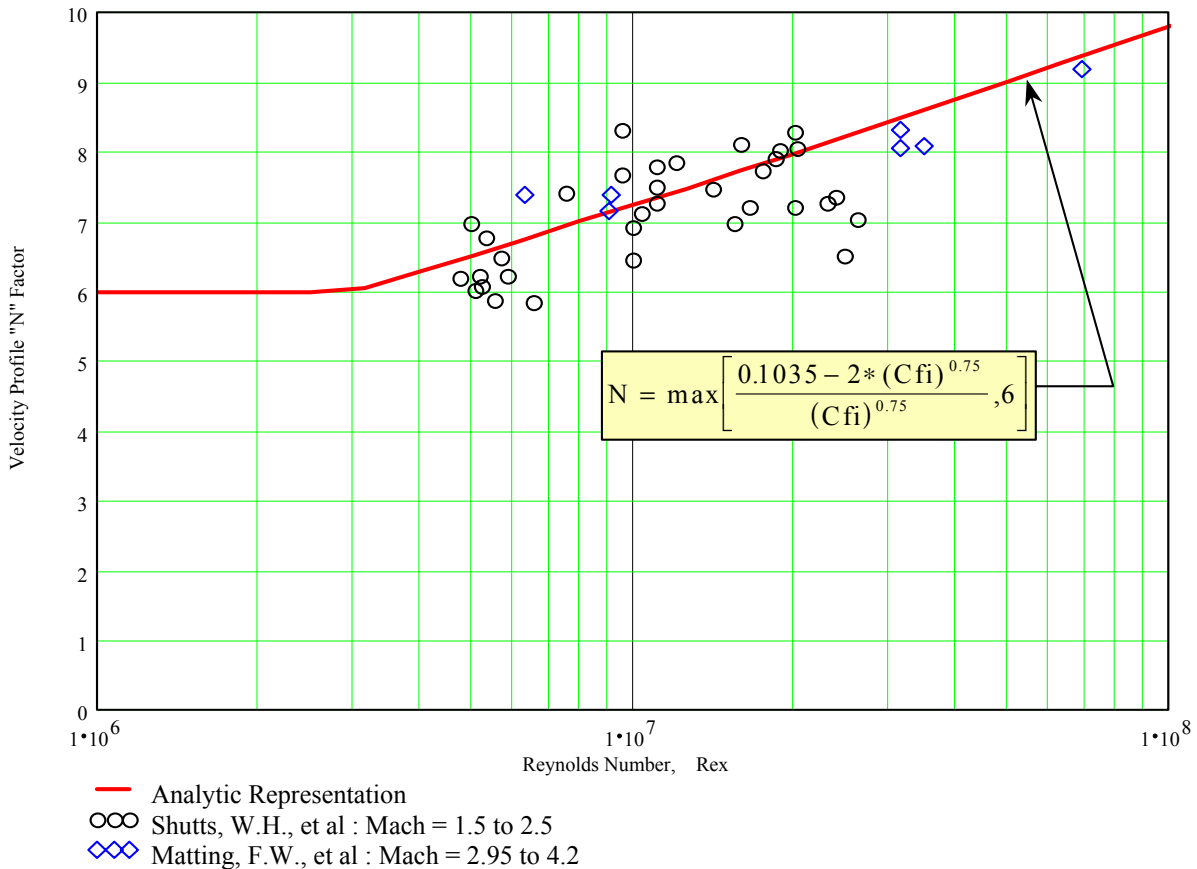


Incompressible velocity profile data from a number of independent sources were used to determine “appropriate” values of  $N$  to represent a turbulent boundary layer. The results as shown in this figure, indicate that the value of “ $N$ ” is strongly dependent on Reynolds number.

The equation shown in the figure was developed in the current study to represent the effect of Reynolds number on “ $N$ ” as determined from the experimental data.

# Supersonic Velocity Profile “N” Factor

$$u/U_o = (y/\delta)^{1/N}$$



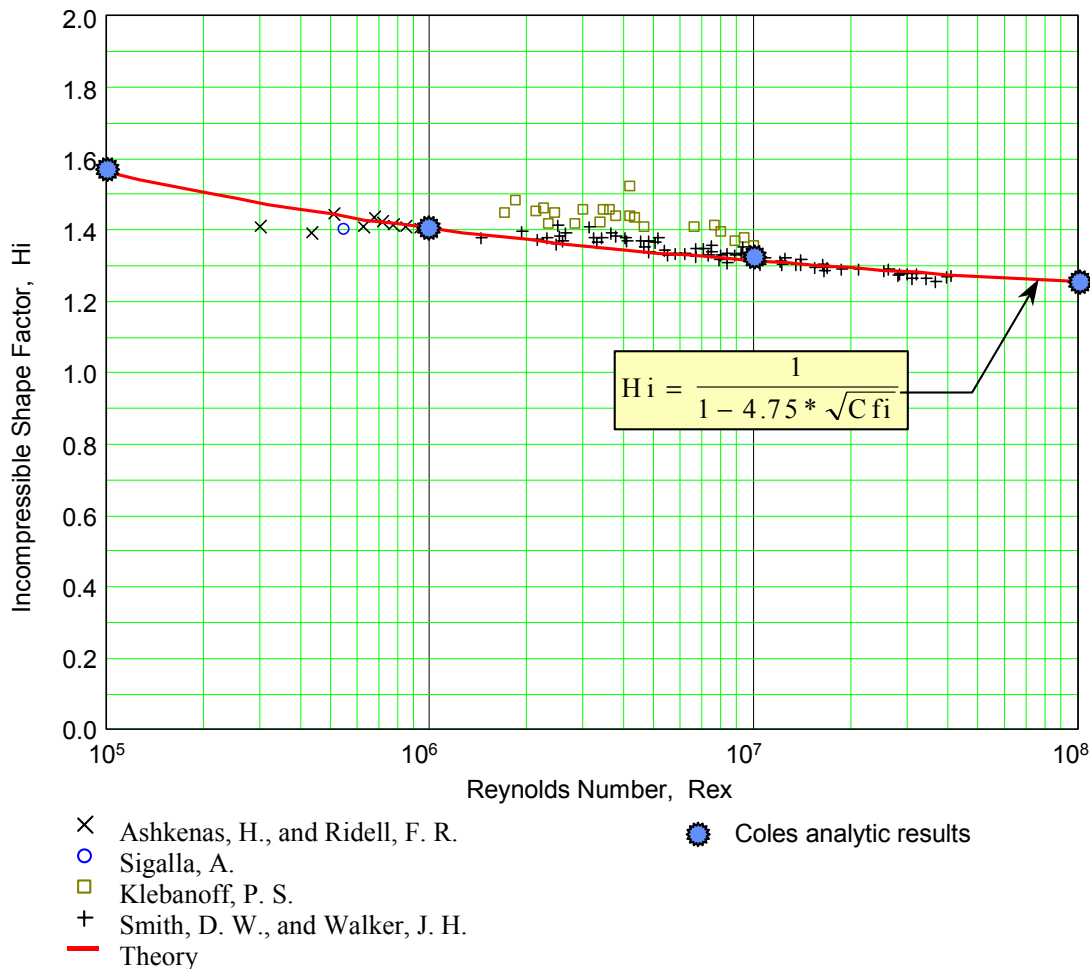
The data in this figure are values of “N” determined from compressible boundary layer measurements for a number of Mach numbers from 1.5 to 4.2. The compressible values of “N” appear to scatter about the empirical equation that was developed from the incompressible velocity profile data.

Thus it appears that the shape of a turbulent boundary layer depends on Reynolds number but is independent of Mach number.

This result should not be surprising for it is implied by the concept of the reference temperature approach to calculate supersonic skin friction drag. Skin friction depends on the shape of the boundary layer as well as the density and viscosity in the boundary. The reference temperature method as defined earlier in this note assumes that compressibility only changes the effective values of density and viscosity. Hence, Mach number doesn’t change the velocity profile shape.

Developing an analytic expression for “N” was the second step in the process for developing a method to predict the boundary layer thickness.

# Incompressible Shape Factor Data



In incompressible flow the boundary layer displacement thickness and momentum thickness equations are:

$$\delta^* = \int_0^{\infty} \left[ 1 - \left( \frac{u}{U_{\infty}} \right) \right] dy$$

$$\theta = \int_0^{\infty} \frac{u}{U_{\infty}} \left[ 1 - \left( \frac{u}{U_{\infty}} \right) \right] dy$$

$$H = \delta^* / \theta$$

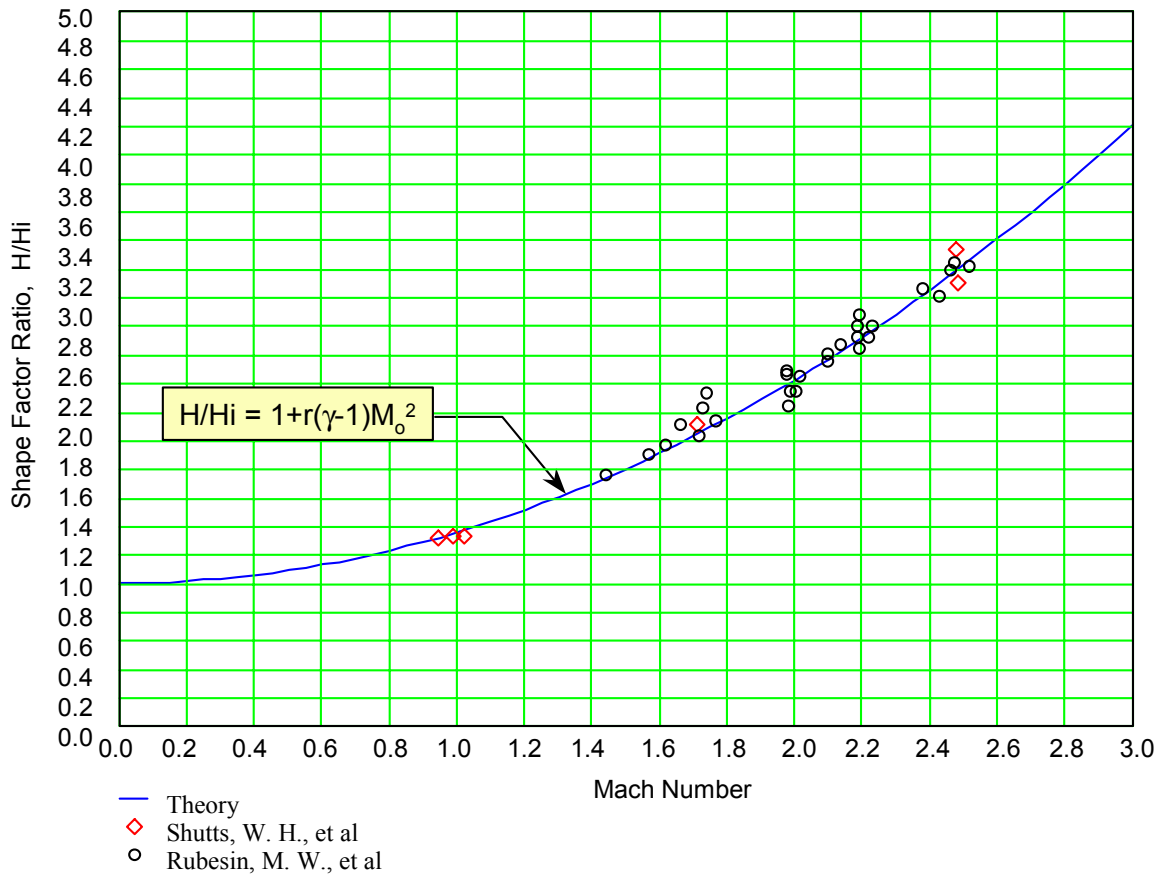
In incompressible flow, the value of H for a flat plate turbulent flow is a unique function of the “shape” of the boundary layer.

In principal, the variation of H with Reynolds number could be determined from the approximate velocity profile shape and the empirical equation for “N”. As previously shown, the approximate velocity profile shape provides a very good estimate for  $\delta^*$ . The values for both  $\theta$  and H calculated using the approximate velocity profile are not as accurate. Consequently, it was decided to use an equation for H developed by Clauser (presented in Ref 28) based on a more sophisticated representation of the boundary layer based on the “velocity defect” concept. Experimental values of the incompressible shape factor,  $H_i$ , are compared with a modified version of Clauser’s equation in which the constant 4.75 replaced Clauser’s original value of 4.31.

Also shown in the figure are analytic values for  $H_i$  calculated by Coles (presented in Ref 35) using “log” wall relations for the boundary layer.

# Mach Number Effect on Boundary Layer Shape Factor

Reynolds Number =  $4 \times 10^6$  to  $30 \times 10^6$



Following Monaghan (Ref 27), the shape factor for the turbulent boundary layer on a flat plate can be related to the incompressible value  $H_i$ , the free stream temperature  $T_\infty$ , the wall temperature  $T_w$ , and the recovery temperature  $T_r$ , by the equation:

$$\frac{H}{H_i} = 1 + \left( \frac{T_w}{T_\infty} - 1 \right) + \left( \frac{T_r}{T_\infty} - 1 \right)$$

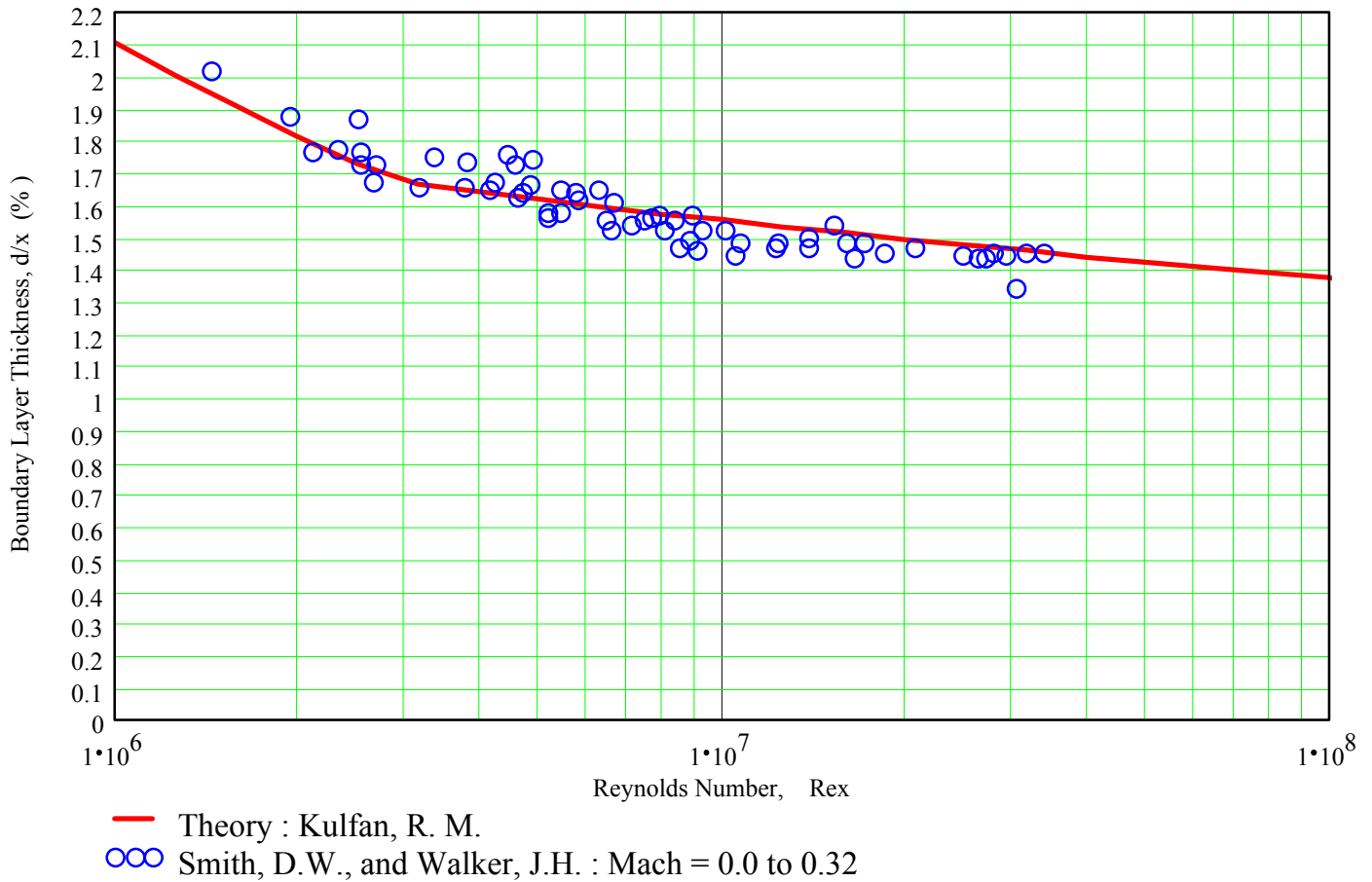
For an insulated surface this equation becomes:

$$\frac{H}{H_i} = 1 + r(\gamma - 1)M_\infty^2$$

Experimental data shown in the figure appears to validate this equation. Hence, the shape factor for fully turbulent flat plate flow can be calculated as the product of two factors. One factor depends only on Reynolds number and the second factor depends only on Mach number.

The equation implies that boundary layer displacement effects become much larger than the momentum thickness as Mach number increases.

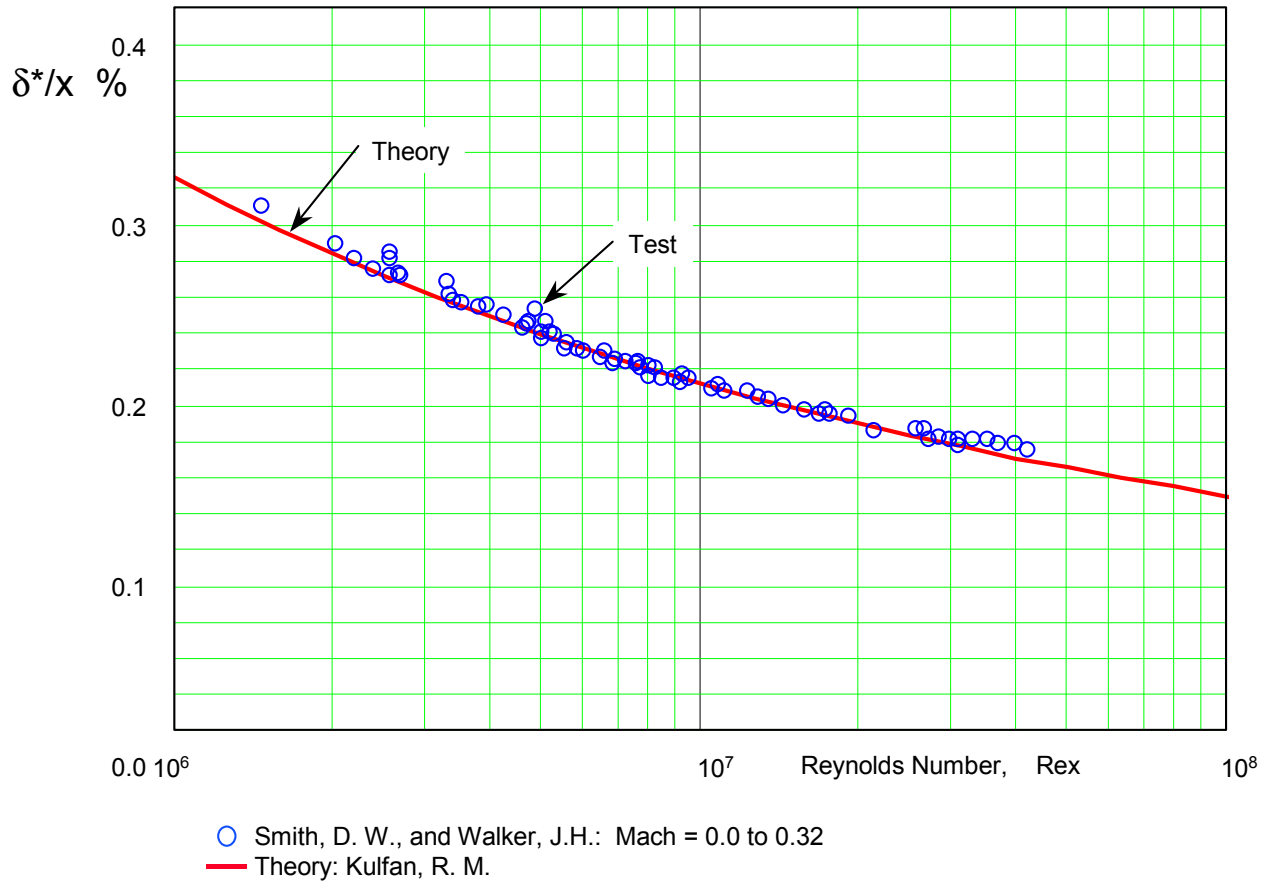
## INCOMPRESSIBLE BOUNDARY LAYER THICKNESS



We now have developed all the ingredients to calculate the boundary layer thickness for fully turbulent flat plate flow using the process shown in figure 23.

Calculations of the variation of incompressible flat plate boundary layer thickness are compared with test data from reference 1. The theoretical predictions obtained using this process closely matches the test data.

# Incompressible Boundary Layer Displacement Thickness

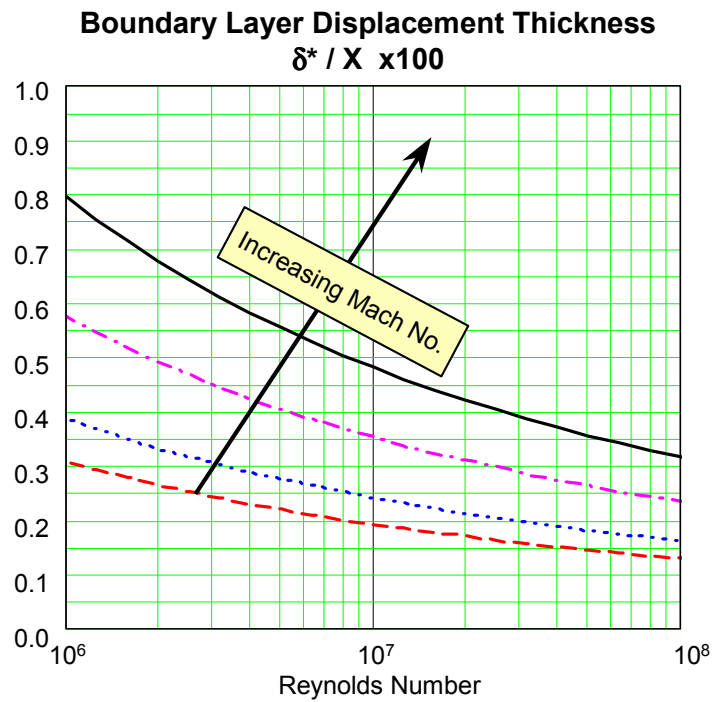
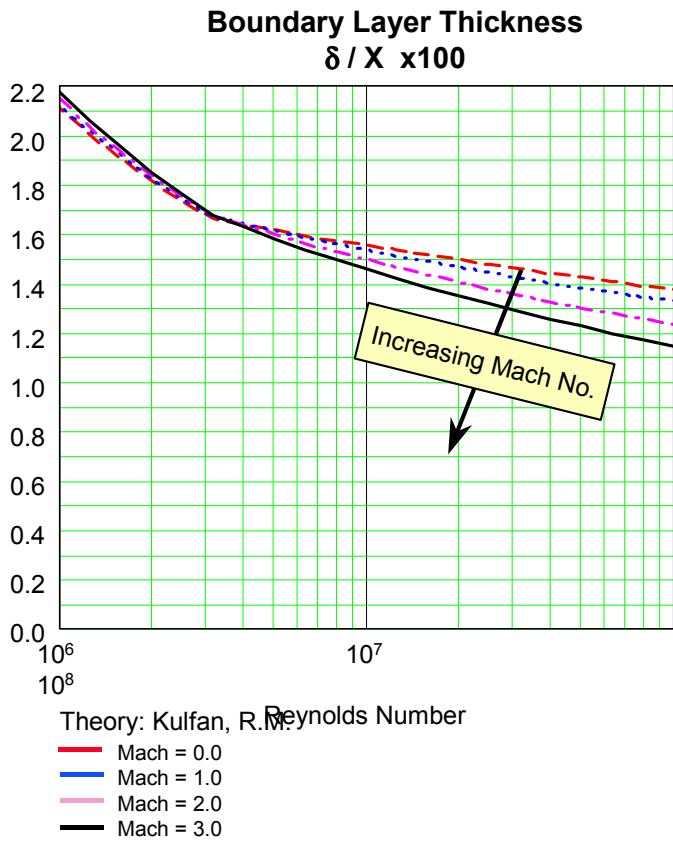


The boundary layer displacement thickness can be calculated by the shape factor equations and the average flat plate skin friction coefficient as:

$$\frac{\delta^*}{X} = \frac{H_i}{2} \frac{H}{H_i} CF$$

Predictions of incompressible displacement thickness are compared with test data in the figure. The agreement between the theory and the test data is quite good.

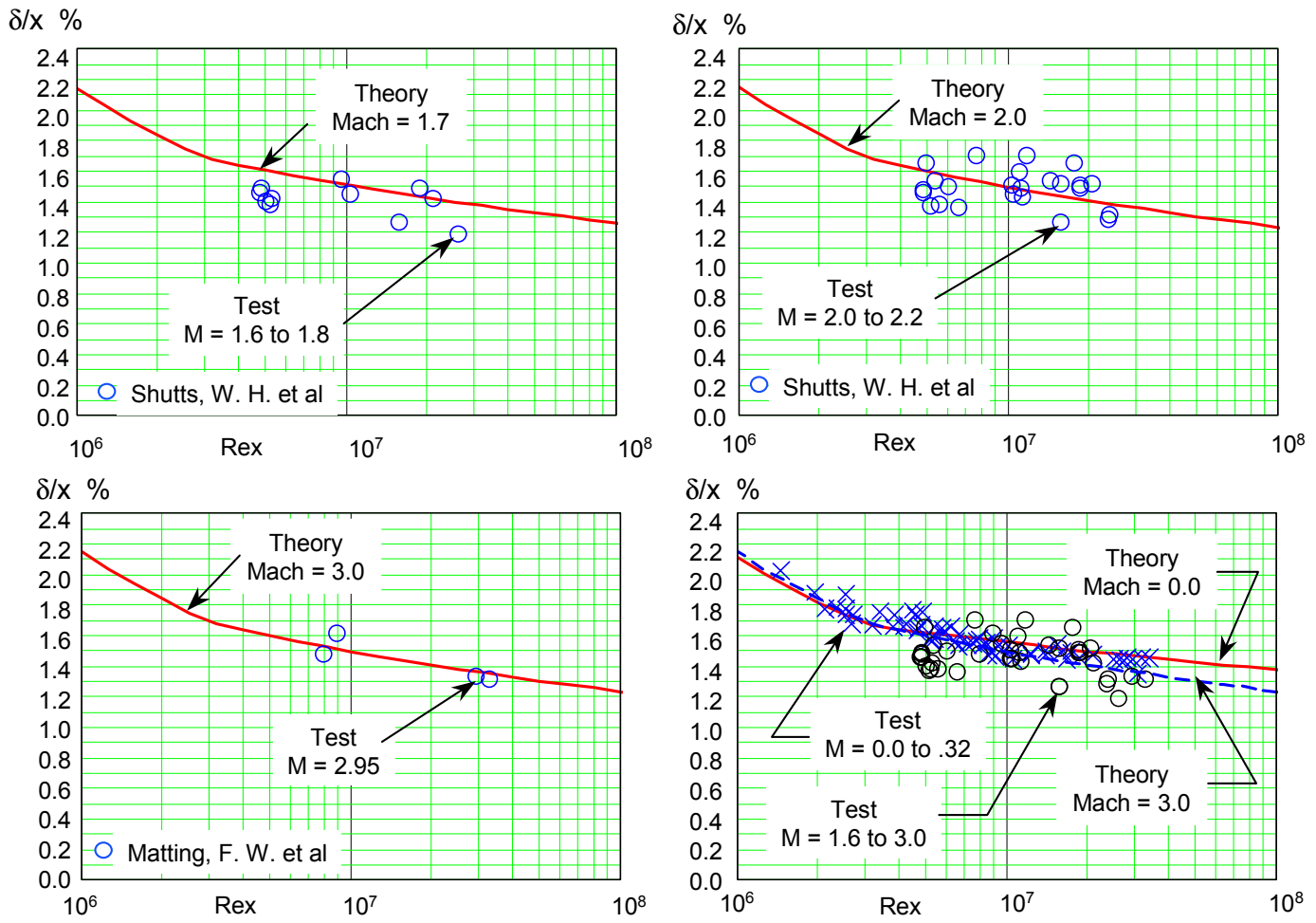
# Compressibility Effects on Boundary Layer Thickness



Boundary layer thickness and displacement thickness have been calculated for a range of Reynolds numbers and Mach numbers from 0 to 3 using the methods presented in this paper. The overall boundary layer thickness is seen to be relatively insensitive to Mach number. The boundary layer displacement thickness, however, grows rapidly as Mach number increases.



# Compressibility Effects on Boundary Layer Thickness



Compressible boundary layer thickness predictions are compared with test data in this Figure for Mach numbers of 1.7, 2.0 and 3.0. Although there is quite a bit of data scatter, the data appears to validate the boundary layer thickness predictions. The fourth figure contains the incompressible data from the previous figure and the three sets of compressible data. This appears to substantiate the conclusion that the thickness of a turbulent boundary layer is indeed relatively insensitive to Mach number.

# Conclusions

- Modified Incompressible Equations and Improved  $T^*/T$  Method  
Predict “Mean” of Available Flat Plate Skin Friction Drag Measurements
- New Methods Presented That Appear to Provide Good Estimates  
of Boundary layer Thickness and Displacement Thickness
- Compressibility Effects Have Very Little Effect on The Shape or Height  
of the Turbulent Flat Plate Velocity Profile.
- Boundary Layer Displacement Thickness Increases Rapidly With Mach Number
- Comparisons of Navier Stokes CFD Predictions of Flat Plate Turbulent Skin  
Friction Drag and Boundary Layer Growth, With the Test Data and / or Theory  
Presented in This paper, is considered to be a Necessary and Vital Step to  
Validating the Codes For HSCT Viscous Drag Predictions .
- Need Additional / Quality Experimental CF Data:
  - Locate Available Existing Data
  - Symmetric Model Tests
  - Segmented Axisymmetry Body of Revolution
  - Utilize TU-144 Flight Test Data
  - ???

The modified incompressible CF equations and the improved  $T^*$  equation presented in the paper appear to consistently match the test better than the other flat plate CF methods currently in use on the HSCT program. It is recommended that the methods presented here, be adapted as the official HSCT flat plate calculation methods.

The boundary layer thickness, and displacement thickness calculations methods presented in this paper seem to be validated by the existing data.

Compressibility effects have little effect on either the shape or height of a turbulent boundary layer. The displacement thickness however varies rapidly with increasing Mach number.

Comparisons of Navier-Stokes predictions, of the skin friction drag and boundary layer growth characteristics for fully turbulent flat plate flow, with the theory and/or test data presented in this paper is considered to be a necessary and vital step in validation of the CFD codes for HSCT viscous drag predictions. This is just the first step in the total validation process that may also include comparisons with data from tests of a symmetric HSCT type configuration, or data from tests of a long segmented cone/cylinder body, and utilization of the newly acquired TU-144 flight test data

# REFERENCES

Each reference listed below contains one or more of the following criteria (as denoted by the letter in the brackets):

- A. Extensive experimental measurement and data reduction techniques.
  - B. Presentation or discussion of analytical techniques.
  - C. Experimental data.
  - D. Extensive bibliography.
- 
1. Kulfan, Robert M., "Turbulent Boundary Layer Flow Past a Smooth Adiabatic Flat Plate", Boeing Document D6-7161, May 1961 [B,C,D]
  2. White, Frank M. VISCOUS FLUID FLOW, Mc Graw - Hill Book Company, 1974, [ B,C,D]
  3. Wegener, Winkler, Sibulkin. "A Measurement of Turbulent Boundary Layer Profiles and Heat Transfer Coefficients at Mach 7.0", Journal of Aero-nautical Sciences, Vol.20, No.3, 1953. [A,C]
  4. Chapman and Kester. "Measurement of Turbulent Skin Friction on Cylinders in Axial Flow at Subsonic and Supersonic Velocities," Journal of Aeronautical Sciences, Vol.20, No.7, 1953. [A,B,C,D]
  5. Chapman and Kester. "Turbulent Boundary Layer and Skin Friction Measurements in Axial Flow along Cylinders at Mach Numbers between 0.5 and 3.6." NACA TN 3097. [A,B,C,D]
  6. Naleid, J. F. "Experimental Investigation of the Impact Probe Method of Measuring Local Skin Friction at Supersonic Speeds in the Presence of an Adverse Pressure Gradient." Defense Research Laboratory Report. D.R.L. - 432, CF - 273. [A,C]
  7. Sibulkin, M. "Boundary Layer Measurements at Supersonic Nozzle Throats." Journal of Aeronautical Sciences, April, 1957. [A,C]
  8. Winkler, E. M., Cha, M. H. "Experimental Investigations of the Effect of Heat Transfer on Hypersonic Turbulent Boundary Layer Skin Friction." Journal of Aero/Space Sciences, February, 1959. [A,C]
  9. Shutts, W. H., Hartwig, W. H., and Weiler, J. E. "Final Report on Turbulent Boundary Layer and Skin Friction Measurements on a Smooth, Thermally Insulated Flat Plate at Supersonic Speeds." Defense Research Laboratory Report. D.R.L. - 364, C.M. - 823. [A,C,D]
  10. Coles, Donald. "Measurements of Turbulent Friction on a Smooth Flat Plate in Supersonic Flow." Journal of Aeronautical Sciences, Vol.21, No.7, July, 1954. [A,C]
  11. Korkegi, Robert H. "Transition Studies and Skin Friction Measurements on an Insulated Flat Plate at a Mach Number of 5.8." Journal of Aeronautical Sciences, Vol.25, No.2, February, 1956. [A,C]

# REFERENCES (Continued)

12. Sommer, Simon C., and Short, Barbara J. "Free Flight Measurements of Skin Friction of Turbulent Boundary Layer Skin Friction in the Presence of Severe Aerodynamic Heating at Mach Numbers from 2.8 to 7.0." NACA TN 3391, March, 1955. [A,B,C]
13. Sommer, Simon C., Short, Barbara J. "Free Flight Measurements of Skin Friction of Turbulent Boundary Layers with High Rates of Heat Transfer at Supersonic Speeds." Journal of Aeronautical Sciences, June, 1956. [A,B,C]
14. Rubesin, M. W., Maydew, R. C., and Varga, S. A. "An Analytical and Experimental Investigation of Skin Friction of the Turbulent Boundary Layer on a Flat Plate at Supersonic Speeds." NACA TN 2305, February, 1951. [A,B,C]
15. Matting, F. We, Chapman, D. R., Nyholm, J. R., Thomas, A. G. "Turbulent Skin Friction at High Mach Numbers and Reynolds Numbers." PROCEEDINGS OF THE 1959 HEAT TRANSFER AND FLUID MECHANICS INSTITUTE. Stanford University Press, 1959. [A,B,C,D]
16. Dhawan, Satish. "Direct Measurements of Skin Friction." NACA Report 1121, 1953. [A,B,C,D]
17. Czarnecki, K. R., Sevier, J. R., and Carmel, N. M. "Effects of Fabrication 'type Roughness on Turbulent Skin Friction at Supersonic Speeds." NACA TN 4299. [A,C]
18. Fenter, F. W., and Stalmach, C. J., Jr. "The Measurement of Local Turbulent Skin Friction at Supersonic Speeds by Means of Surface Impact Pressure Probes." Defense Research Laboratory Report. D.R.L. - 392, CM - 878, October, 1957. [A,B,C,D]
19. Kuethe, A.M., and Schetzer, J. D. , FOUNDATIONS OF AERODYNAMICS John Wiley and Sons, Incorporated, . New York, 1959. [B]
20. Nielsen, J. N. MISSILE AERODYNAMICS. New York, McGraw-Hill Book Company Inc., 1960. [B,C]
21. Liepmann H. W., and Roshko, A., ELEMENTS OF GASDYNAMICS, John Wiley and Sons, Inc., New York 1957. [B,D]
22. Shapiro, A. H. THE DYNAMICS AND THERMODYNAMICS OF COMPRESSIBLE Flow. Vol.11, New York, The Ronald Press Company, 1954. [B,C]
23. Schlichting, H. BOUNDARY LAYER THEORY. New York, Pergamon Press, 1955. [B,D]
24. Hilsenrathb J. et al. TABLES OF THERMAL PROPERTIES OF GASES. National Bureau of Standards Circular 564, November, 1955.
25. Smith, D. W., and Walker, J. H. "Skin Friction Measurements in Incompressible Flow." NACA TN 4231, 1958. [A,C]

# REFERENCES (Continued)

26. Matting, F. W., Chapman, D. R., Nyholm, J. R., Thomas, A. G. "Turbulent Skin Friction at High Mach Numbers and Reynolds Numbers in Air and Helium." NASA R-82. [A,B,C,D]
27. Monaghan, R. J. "On the Behavior of Boundary Layers at Supersonic Speeds." I.A.S. - R A. S Proceedings, 1955. [B,D]
28. Lin, C. C. TURBULENT FLOWS AND HEAT TRANSFER. Vol.5, Princeton Series on High Speed Aerodynamics and Jet Propulsion. Princeton University press, 1959. [B,D]
29. Thwaites, B. "Incompressible Aerodynamics." Oxford at the Clarendon Press, 1960. [B,D]
30. Klebanoff, P.S., and Diehl, Z. E. "Some Features of Artificially Thickened Fully Developed Boundary Layers with Zero Pressure Gradient." NACA Report 1110, 1952. [A,C,D]
31. Simon, P. C. and Kowalski, K. L. "Charts of Boundary Layer Mass Flow and Momentum for Inlet Performance Analysis Mach Number Range, 0.2 to 5.0." NACA TN 3583, November, 1955. [B]
32. Ashkenas, H., and Riddell, F. R. "Investigation of the Turbulent Boundary Layer on a Yawed Flat Plate." NACA TN 3383, 1955. [A,B,C]
33. Klebanoff, P.S. "Characteristics of Turbulence in a Boundary Layer with Zero Pressure Gradient." NACA Report 1247, 1955. [A,B,C]
34. Sigalla, A. "Experiments with Pitot Tubes for Skin Friction Measurements." British Iron and Steel Research Association Report P/2/58, List 92, 1958. [A,C]
35. Duncan, W. J., Thom, A. S., Young, A. D., MECHANICS OF FLUIDS, American Elsevier Publishing Co. New York, 1970. [A,B,C,D]
36. Erickson, W. D. "Real-Gas Correction Factors for Hypersonic Flow Parameters in Helium." NASA TN D-462, September, 1960.
37. Eckert, E. R. G. et al: "Prandtl Number, Thermal Conductivity, and Viscosity of Air-Helium Mixtures." NASA TN D-533, September, 1960.
38. Czarnecki, K. R. et al: "Investigation of Distributed Surface Roughness on a Body of Revolution at a Mach Number of 1.61." NACA TN 3230, June, 1954. [A,C]
39. Smith, D. W., and Walker, J. H. "Skin Friction Measurements in Incompressible Flow." NACA TR R-26, 1959. [A,C,D]
40. Eckert, Z. R. G. "Simplified treatment of the Turbulent Boundary Layer along a Cylinder in Compressible Flow." Journal of Aeronautical Sciences, January, 1952. [B]